

VIBRATION OF AN ELL-SHAPED UNIFORM
FILAMENT WITH CLAMPED ENDS

By

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THESIS

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December 1970

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Vibration of an Ell-shaped Uniform
Filament with Clamped Ends

by

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ABSTRACT

An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotary inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.

Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.

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I. INTRODUCTION

A. GENERAL

In order to assure long time integrity of nuclear piping, the applicable codes, ANSI B31.7 - 1969 Code for Nuclear Power Piping and forthcoming edition of Section III of the ASME Boiler and Pressure Vessel Code, call for seismic analysis of such piping and require that the severity of the effects of seismically induced vibration be maintained below specified acceptable levels. Generally the problem of vibration analysis of the major piping systems is so complex that a major computational effort is required and this demands digital computer analysis using quite sophisticated programs. However, in the early stages of a design, when alternate layouts are being investigated, it is useful to be able to make some reasonable estimates of what a more elaborate analysis would be likely to reveal.

A recent document [3] concerned with assisting a designer to understand and comply with ANSI B31.7 contains material summarizing the vibration characteristics of several simple piping configurations. It suggests how the designer might employ this sort of information to assist him in estimating the vibration characteristics of more complicated configurations. This material should prove to be quite useful.

However, the scope of the configurations about which such information is available is quite limited. One of the

basic configurations with which a designer deals is a simple ell-shaped configuration. It was decided that it would be quite useful to determine and present in a concise form the significant results relating to the vibration of such a configuration. Specifically, it is assumed that the two ends of the configuration are perfectly fixed or clamped, that the pipe is of uniform properties along its entire length and is of rather small diameter compared to its length (hence the word "filament" in the title of this thesis) so that shear deflection and rotatory inertia could be neglected, and that the right angle junction of the two legs of the ell is small compared to the overall dimensions of the ell itself, so that it is not necessary to consider the properties of the bend or elbow that would exist in practice at the junction.

Such results are obtained and given herein. The basic parameter is the ratio of the length of the shorter leg to that of the longer leg; this parameter is designated by the Greek letter rho in what follows. A second important parameter, Greek beta in what follows, is required to relate the coupling that takes place between bending and torsion in the case of what are called out-of-plane vibrations.

This simple ell-shaped configuration has the great virtue of being such that the vibration problem is naturally divided into two disjoint sub-parts. One such part involves only displacements which lie in the plane of the ell; no torsion is involved in these cases. The other part involves

both bending and torsion, but the motion of any point on the axis of the filament is only perpendicular to the plane of the ell. This gives rise to the two terms used in what follows: "in-plane vibrations" and "out-of-plane vibrations."

B. ASSUMPTIONS AND LIMITATIONS

It is well to repeat here specifically the assumptions and limitations involved in this investigation.

1. The configuration is formed from originally straight uniform elastic members which meet at a right angle.
2. The angle at which the two members meet remains a right angle throughout the deformation of the configuration.
3. The ends of the configuration are assumed to be perfectly clamped.
4. The effect of shear deformation has been neglected. Only bending and torsional compliances are considered and deformations are assumed to be "small."
5. Rotatory inertia has been neglected.
6. Poisson's ratio has been taken to be 0.3.

Additionally, there are limitations implicit in the fact that values of the roots of the equations which are later solved are obtained only to an accuracy of one part in one million parts and in the fact that the computational programs used a mantissa having 56 bits. These limitations resulted in being able to obtain only about the lowest

thirteen frequencies for a particular configuration. Also, only a few values of β were investigated and there are some difficulties in the use of linear interpolation for other values of β .

C. NOTATION

A, B, C, D, F, H Constants in equation (2-5) and (2-14)

E Modulus of elasticity

G Shear modulus of elasticity

g_1, g_2, g_3, g_4 Functions defined in equation (2-6)

I Moment of inertia of cross section

J Polar moment of inertia

L Length of pipe

M Bending moment

P Defined in equation (2-4)

t Time

T Torque

V Shear force

x Distance from fixed end

y Lateral deflection

Y Lateral deflection mode shape

Y' Slope

α Defined in equation (2-13)

β Defined in equation (2-29)

γ Phase angle in equation (2-11)

θ Angle of twist

ϕ Angle of twist mode shape

ψ	Constant in equation (2-2)
μ	Mass per unit length
ν	Mass moment of inertia per unit length
ρ	Length ratio
ω	Circular frequency

II. THEORETICAL DEVELOPMENTS

A. BENDING ANALYSIS

Neglecting the effects of shear deflection and rotatory inertia the equation governing lateral vibration of a uniform beam is

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = 0 \quad (2-1)$$

where μ is mass per unit length, EI is flexural rigidity, y is lateral deflection and t is time. Assuming harmonic oscillation with circular frequency ω , the solution of equation (2-1) can be written in the following form:

$$y(x,t) = Y(x) \cos(\omega t + \psi) \quad (2-2)$$

where ψ is a constant and Y is a function of x alone. Substitution of equation (2-2) into equation (2-1) yields

$$\frac{d^4 Y(x)}{dx^4} - P^4 Y(x) = 0 \quad (2-3)$$

where

$$P^4 = \frac{\mu \omega^2}{EI} \quad (2-4)$$

The general solution of equation (2-3) may be written as

$$Y(x) = Ag_1 + Bg_2 + Cg_3 + Dg_4 \quad (2-5)$$

where

$$g_1 = \frac{1}{2}[\cosh(Px) + \cos(Px)] \quad (a)$$

$$g_2 = \frac{1}{2}[\sinh(Px) - \sin(Px)] \quad (b)$$

(2-6)

$$g_3 = \frac{1}{2}[\cosh(Px) - \cos(Px)] \quad (c)$$

$$g_4 = \frac{1}{2}[\sinh(Px) + \sin(Px)] \quad (d)$$

At any location x , deflection Y_x , slope Y'_x , bending moment M_x , and shear force V_x can be expressed as

$$Y_x = Ag_1 + Bg_2 + Cg_3 + Dg_4 \quad (a)$$

$$Y'_x = P(Ag_2 + Bg_3 + Cg_4 + Dg_1) \quad (b)$$

(2-7)

$$M_x = EIP^2(Ag_3 + Bg_4 + Cg_1 + Dg_2) \quad (c)$$

$$V_x = EIP^3(Ag_4 + Bg_1 + Cg_2 + Dg_3) \quad (d)$$

At $x = 0$,

$$Y_0 = A \quad ; \quad Y'_0 = DP \quad (a,b)$$

(2-8)

$$M_0 = C(EIP^2) \quad ; \quad V_0 = B(EIP^3) \quad (c,d)$$

Expressing the constants A , B , C and D in terms of Y_0 , Y'_0 , M_0 and V_0 and substituting into equation (2-7) yields:

$$\begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_x = \begin{bmatrix} g_1 & g_4/P & g_3/EIP^2 & g_2/EIP^3 \\ Pg_2 & g_1 & g_4/EIP & g_3/EIP^2 \\ EIP^2g_3 & EIPg_2 & g_1 & g_4/P \\ EIP^3g_4 & EIP^2g_3 & Pg_2 & g_1 \end{bmatrix} \begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_0 \quad (2-9)$$

The above equation expresses the deflection, slope, bending moment, and shear force at any location x in terms of those at $x = 0$.

B. TORSION ANALYSIS

The equation of motion of a uniform cylinder undergoing torsional vibration is,

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{v}{JG} \frac{\partial^2 \theta}{\partial t^2} = 0 \quad (2-10)$$

where v is mass moment of inertia per unit length, J is polar moment of inertia of elastic resistance, and θ is angle of twist. For a sinusoidal solution at circular frequency ω ,

$$\theta(x,t) = \phi(x) \cos(\omega t + \gamma) \quad (2-11)$$

where γ is a constant and ϕ is a function of x alone. Substitution in equation (2-10) gives

$$\frac{d^2 \phi(x)}{dx^2} + \alpha^2 \phi(x) = 0 \quad (2-12)$$

where

$$\alpha^2 = \frac{v\omega^2}{JG} \quad (2-13)$$

The general solution of equation (2-12) is

$$\phi(x) = F \cos(\alpha x) + H \sin(\alpha x) \quad (2-14)$$

and torque T is

$$T = JG \frac{d\phi(x)}{dx} = JG\alpha [-F \sin(\alpha x) + H \cos(\alpha x)] \quad (2-15)$$

At $x = 0$

$$\phi_0 = F \quad ; \quad T_0 = H(JG\alpha) \quad (2-16a,b)$$

Expressing constants F and H in terms of ϕ_0 and T_0 and substituting into equations (2-14) and (2-15), the angle of twist ϕ_x and torque T_x at any location x , can be expressed in terms of T_0 and ϕ_0 as follows:

$$\begin{bmatrix} \phi \\ T \end{bmatrix}_x = \begin{bmatrix} \cos(\alpha x) & \sin(\alpha x)/JG\alpha \\ -JG\alpha \sin(\alpha x) & \cos(\alpha x) \end{bmatrix} \begin{bmatrix} \phi \\ T \end{bmatrix}_0 \quad (2-17)$$

C. IN-PLANE AND OUT-OF-PLANE VIBRATION ANALYSIS OF AN ELL-SHAPED UNIFORM FILAMENT WITH CLAMPED ENDS

1. In-plane Vibrations

In Figure 2-1, an ell-shaped configuration and the corresponding free body diagrams are shown.

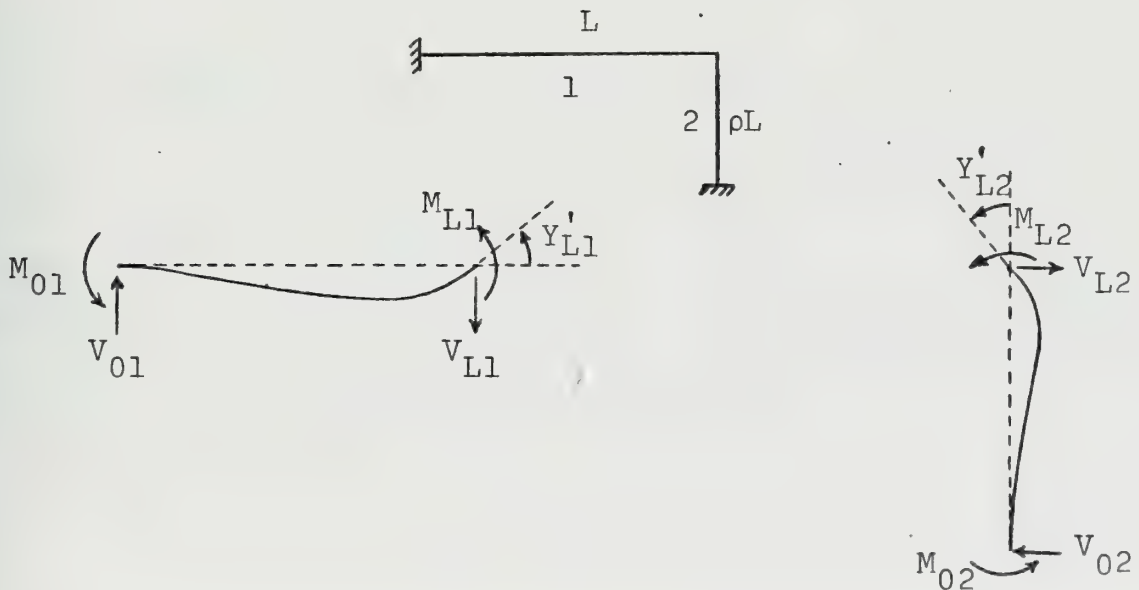


Figure 2-1. Ell-shaped Configuration and Free Body Diagrams for In-plane Vibrations.

The ratio, ρ , of the length of the shorter leg to that of the longer is a basic geometrical parameter in the following analysis.

At the clamped ends deflections Y_{01} , Y_{02} and slopes Y'_{01} , Y'_{02} are equal to zero. Also at the junction of the two portions, deflections Y_{L1} and Y_{L2} are equal to zero. Application of equation (2-9) to the two portions yields,

$$\frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} = 0 \quad (a)$$

$$\frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} = Y'_{L1} \quad (b)$$

$$g_1 M_{01} + \frac{g_4}{P} V_{01} = M_{L1} \quad (c)$$

(2-18)

$$\frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} = 0 \quad (d)$$

$$\frac{\bar{g}_4}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^2} V_{02} = Y'_{L2} \quad (e)$$

$$\bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} = M_{L2} \quad (f)$$

where g_i indicates evaluation of $g_i(x)$ for argument (PL) and \bar{g}_i indicates evaluation for argument (ρPL).

At the junction,

$$Y'_{L1} = Y'_{L2} \quad ; \quad M_{L1} + M_{L2} = 0 \quad (2-19a,b)$$

Combining equations (2-19) with equations (2-18), the following set of equations is obtained.

$$\begin{bmatrix} g_3/EIP^2 & 0 & g_2/EIP^3 & 0 \\ 0 & \bar{g}_3/EIP^2 & 0 & \bar{g}_2/EIP^3 \\ g_4/EIP & -\bar{g}_4/EIP & g_3/EIP^2 & -g_3/EIP^2 \\ g_1 & \bar{g}_1 & g_4/P & \bar{g}_4/P \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \quad (2-20)$$

For a nontrivial solution, the determinant, D, of the above matrix must vanish. Elementary manipulations on D lead to the following form.

$$\begin{vmatrix} g_3 & 0 & g_2 & 0 \\ 0 & \bar{g}_3 & 0 & \bar{g}_2 \\ g_4 & -\bar{g}_4 & g_3 & -\bar{g}_3 \\ g_1 & \bar{g}_1 & g_4 & \bar{g}_4 \end{vmatrix} = 0 \quad (2-21)$$

The terms of equation (2-21) are functions of ρ . For any value of ρ , the zeros $(PL)_i$ of the determinant can be found. The corresponding frequencies are evaluated by using equation (2-4) as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L^4}} \quad (2-22)$$

Solution of equation (2-21) will be explained in Section III-A.

2. Out-of-plane Vibrations

Out-of-plane vibrations can be analyzed similarly. However in this case the effect of torsion must be taken into account.

In Figure 2-2, free body diagrams of an ell-shaped configuration for out-of-plane vibrations are shown.

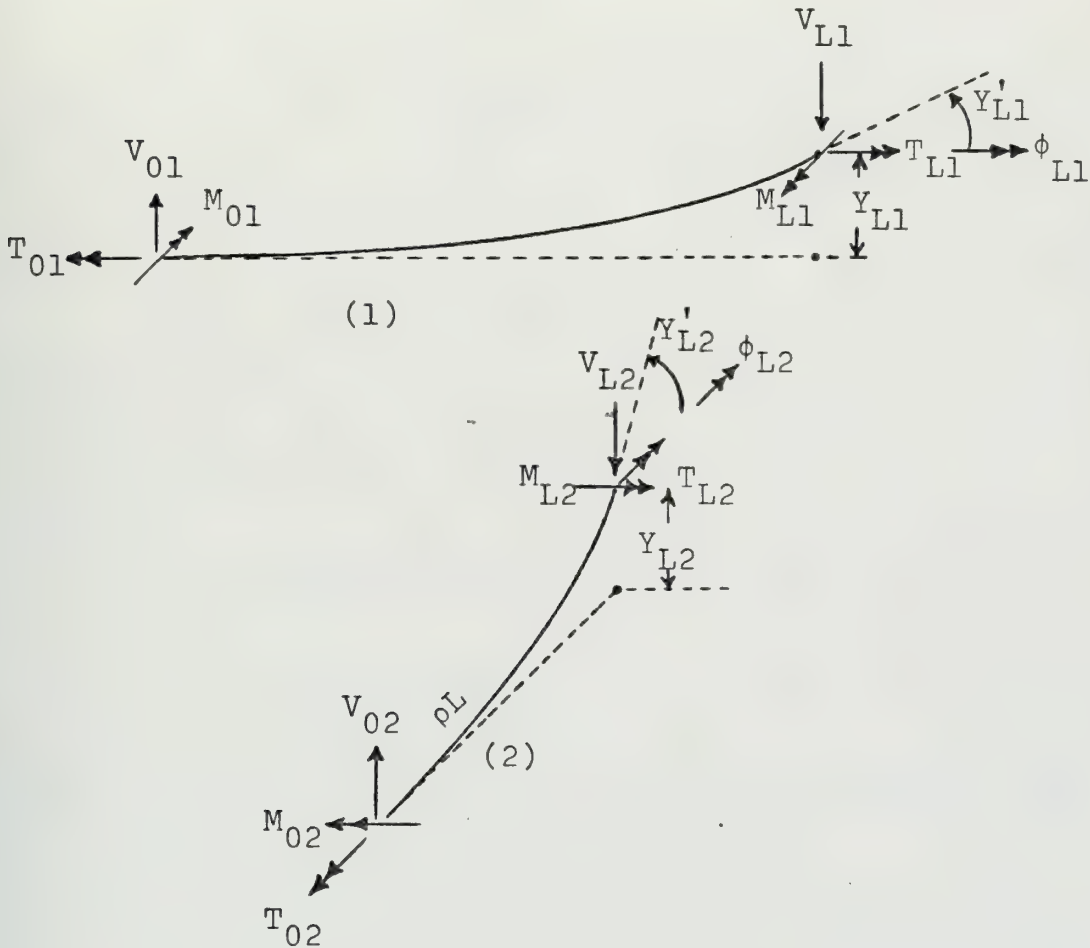


Figure 2-2. Free Body Diagrams of an Ell-shaped Configuration for Out-of-plane Vibrations.

At the clamped ends deflections Y_{01} , Y_{02} and slopes Y'_{01} , Y'_{02} and angles of twist ϕ_{01} , ϕ_{02} are equal to zero. Application of equation (2-9) to the two portions yields

$$Y_{L1} = \frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} \quad (a)$$

$$Y'_{L1} = \frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} \quad (b)$$

$$M_{L1} = g_1 M_{01} + \frac{g_4}{P} V_{01} \quad (c)$$

$$V_{L1} = P g_2 M_{01} + g_1 V_{01} \quad (d)$$

$$Y_{L2} = \frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} \quad (e) \quad (2-23)$$

$$Y'_{L2} = \frac{\bar{g}_4}{EIP} M_{02} + \frac{\bar{g}_3}{EIP^2} V_{02} \quad (f)$$

$$M_{L2} = \bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} \quad (g)$$

$$V_{L2} = P \bar{g}_2 M_{02} + \bar{g}_1 V_{02} \quad (h)$$

where g_i indicates evaluation of $g_i(x)$ for argument (PL) and \bar{g}_i indicates evaluation for argument (pPL).

For the torsion, equation (2-17) applies and gives,

$$\phi_{L1} = \frac{\sin(\alpha L)}{JG\alpha} T_{01} \quad (a)$$

$$T_{L1} = \cos(\alpha L) T_{01} \quad (b)$$

$$\phi_{L2} = \frac{\sin(\alpha p L)}{JG\alpha} T_{02} \quad (c) \quad (2-24)$$

$$T_{L2} = \cos(\alpha p L) T_{02} \quad (d)$$

At the junction:

$$M_{L1} = T_{L2} \quad ; \quad T_{L1} = -M_{L2} \quad (a,b)$$

$$Y'_{L1} = -\phi_{L2} \quad ; \quad \phi_{L1} = Y'_{L2} \quad (c,d) \quad (2-25)$$

$$V_{L1} = -V_{L2} \quad ; \quad Y_{L1} = Y_{L2} \quad (e,f)$$

Specializing to the case of circular cross section and assuming Poisson's ratio is 0.3, then

$$\frac{1}{JG\alpha} = \frac{1.3}{EI\alpha} \quad (2-26)$$

Combination of equations (2-23), (2-24), (2-25) and (2-26) gives the following set of equations.

$$\begin{bmatrix} g_3^P & -\bar{g}_3^P & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4^P & -\frac{1.3P^2}{\alpha}\sin(\alpha L) & 0 & 0 & \bar{g}_3 \\ g_4^P & 0 & 0 & \frac{1.3P^2}{\alpha}\sin(\alpha L) & g_3 & 0 \\ g_2^P & \bar{g}_2^P & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & -\cos(\alpha L) & \frac{g_4}{P} & 0 \\ 0 & \bar{g}_1 & \cos(\alpha L) & 0 & 0 & \frac{\bar{g}_4}{P} \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ T_{01} \\ T_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \quad (2-27)$$

For a nontrivial solution, the determinant D of this matrix must vanish. From equations (2-4), (2-13) and (2-26) it is found that

$$(\alpha L) = \beta(PL)^2 \quad (2-28)$$

where

$$\beta = \sqrt{\frac{1.3v}{\mu L^2}} \quad (2-29)$$

is a dimensionless parameter. Substituting equation (2-28) into equation (2-27) and performing elementary manipulations on D give the following equation

$$\begin{vmatrix} g_3 & -\bar{g}_3 & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4 & -\frac{1.3}{\beta} \sin[\beta(PL)^2] & 0 & 0 & g_3 \\ g_4 & 0 & 0 & \frac{1.3}{\beta} \sin[\rho\beta(PL)^2] & g_3 & 0 \\ g_2 & \bar{g}_2 & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & (PL)\cos[\beta\rho(PL)^2] & g_4 & 0 \\ 0 & \bar{g}_1 & (PL)\cos[\beta(PL)^2] & 0 & 0 & \bar{g}_4 \end{vmatrix} = 0 \quad (2-30)$$

The terms of equation (2-30) are functions of ρ and β . For given values of ρ and β , the zeros $(PL)_i$ of determinant D can be found. The corresponding frequencies are evaluated by using equation (2-22).

Solution of equation (2-30) will be explained in Section III-A.

III. PRESENTATION OF RESULTS

A. METHODS OF SOLUTION

The equations (2-21) and (2-30), developed in Chapter II, can be solved either by algebraic manipulations or by using direct computer solution.

In a first approach to a solution, algebraic manipulations were performed to reduce the order of determinants. Then transcendental equations were obtained and two computer programs, one for the in-plane solution and one for the out-of-plane solution, were developed to get the roots of these equations.

In the computer programs the transcendental equations were used as function subprograms and roots located in an interval. Then the Newton-Raphson method was used to obtain good approximate values of the roots.

The procedure explained above led to some difficulties. During the solution of the equation which was derived from equation (2-30), the Newton-Raphson method failed to give an expected root because the slope of the transcendental function reached very high values and this caused a jump to adjacent roots. In one case, the number of iterations required exceeded three hundred with an allowable error of 10^{-8} (for the quantity PL) and increasing the allowable error to 10^{-6} did not make much difference. Finally, comparison of results with the classical solution in the case ρ is

equal to zero showed that some of the roots were extraneous.

The difficulties mentioned in the preceding paragraph were eliminated by making direct computer evaluation of the determinants (2-21) and (2-30). For this purpose subroutine BRODE was developed and each element of the determinants was stored in this subprogram. In addition to subroutine BRODE, library subroutine DTERM which was developed by Jean Bow in 1967 was used to evaluate the determinants. This modified procedure resulted in reduction of computer time required and in the elimination of extraneous roots.

Difficulties with occasional failure of the Newton-Raphson method were reduced by using the method of successive bisectioning. In this method the number of iterations required for one root did not exceed fifty with an allowable error of 10^{-6} . This procedure also resulted in conservation of computer time.

Detailed instructions for use of the computer programs are given in Appendix B.

B. GRAPHS

Numerical results given to a large number of significant figures were obtained primarily for the purpose of investigating the accuracy and integrity of the solutions. Chapter IV will give a discussion of these investigations.

However, so as to present the results in the most convenient form for practical use, graphical outputs

were also obtained by use of the library subroutine DRAW.

One graph was obtained for the in-plane solution. For the out-of-plane solution five graphs were obtained corresponding to five different values of β . In each case, the ordinates are values of (PL) and abscissas are values of ρ . On these graphs the first (lowest) curve is for the first mode, the second curve for the second mode and so on. Graphs are given in Figures 3-1 through 3-6.

C. SAMPLE PROBLEM

A sample problem will be solved to show how to use the graphs given in the preceding article.

Properties of ell-shaped configuration:

L_1	:	16' - 6"
L_2	:	10' - 3½"
Outside diameter of pipe	:	6.625"
Inside diameter of pipe	:	6.065"
Wall thickness of pipe	:	0.280"
Weight of pipe	:	18.98 lb/ft
Thickness of insulation	:	1 1/8"
Weight of insulation	:	2.90 lb/ft
Weight of contents	:	12.51 lb/ft
Modulus of elasticity E	:	27,400,000 lb/in ²
G	:	10,600,000 lb/in ²

For the use of graphs, values of ρ and β must be known. For this problem,

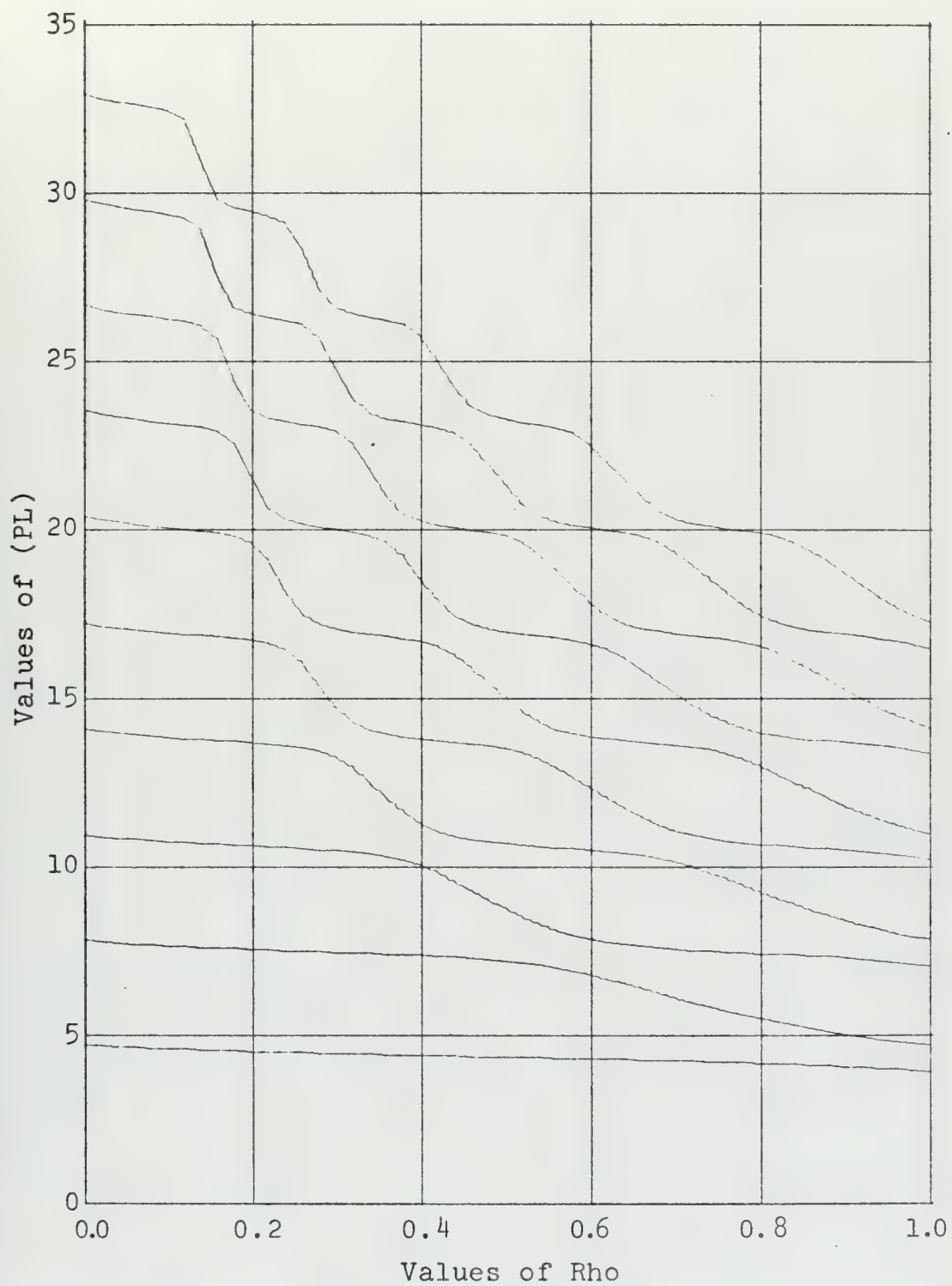


Figure 3-1. In-plane Vibrations.

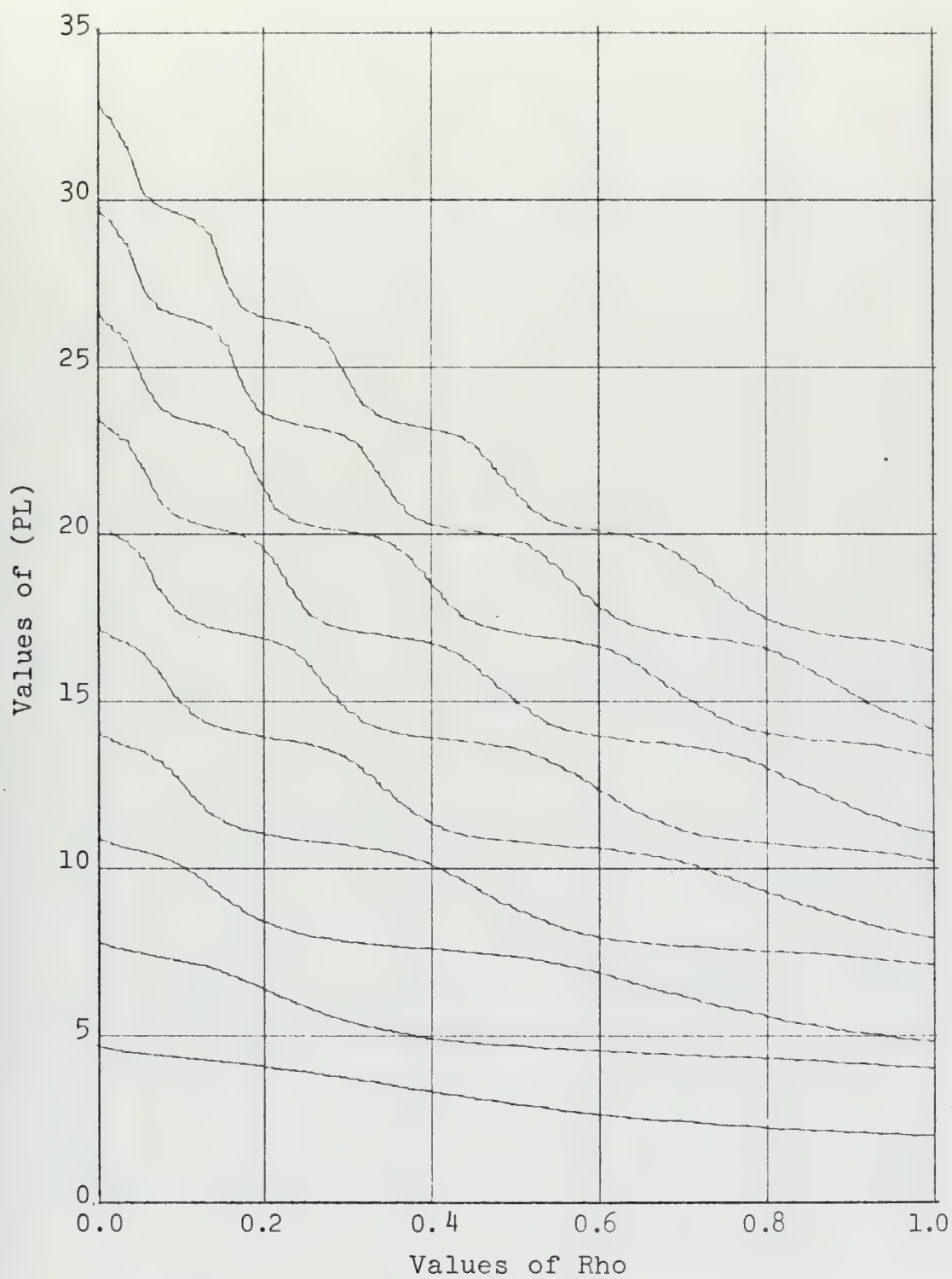


Figure 3-2. Out-of-plane Vibrations, $\text{Beta} = 0.00001$.

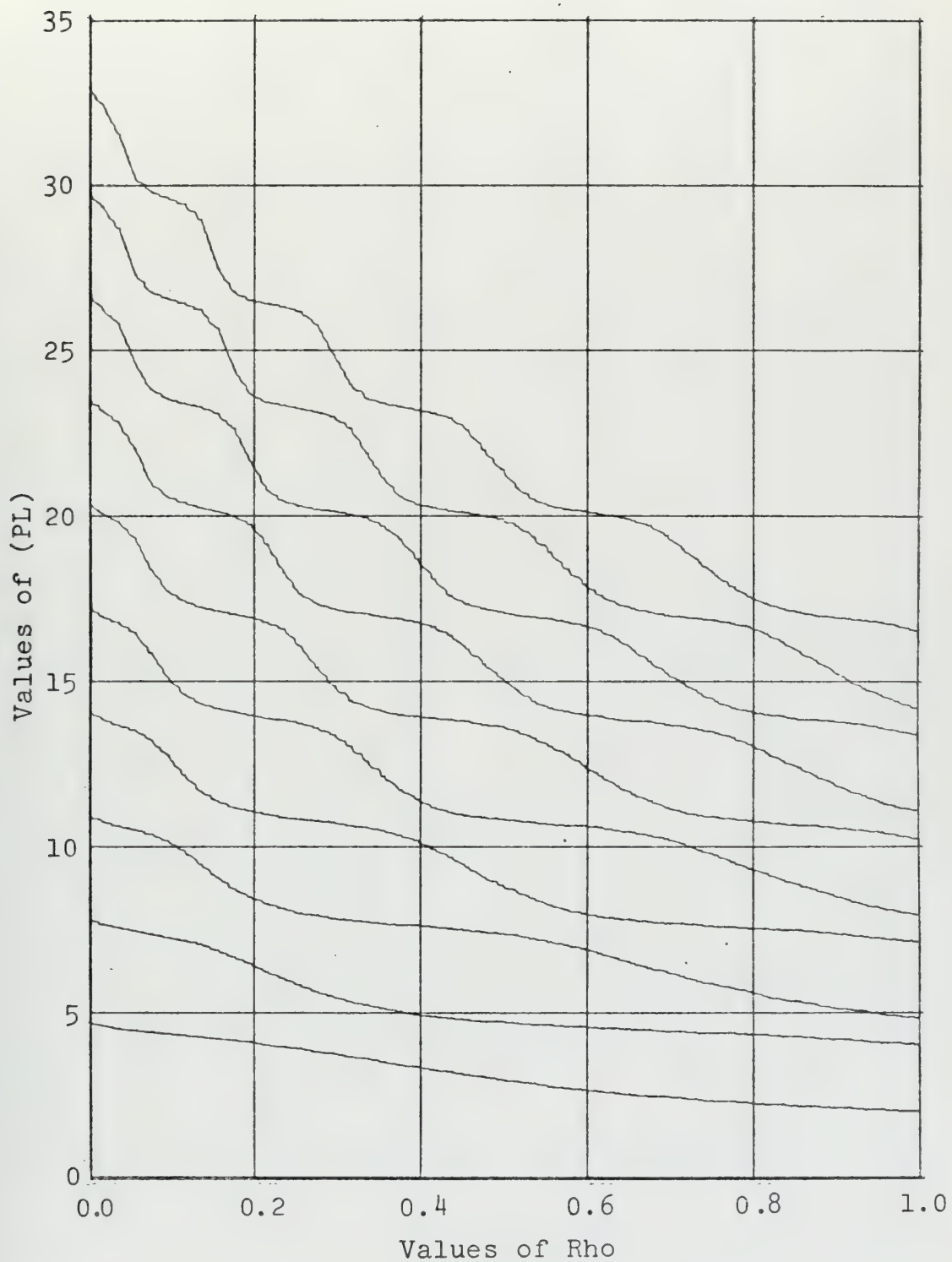


Figure 3-3. Out-of-plane Vibrations, $\beta = 0.001$.

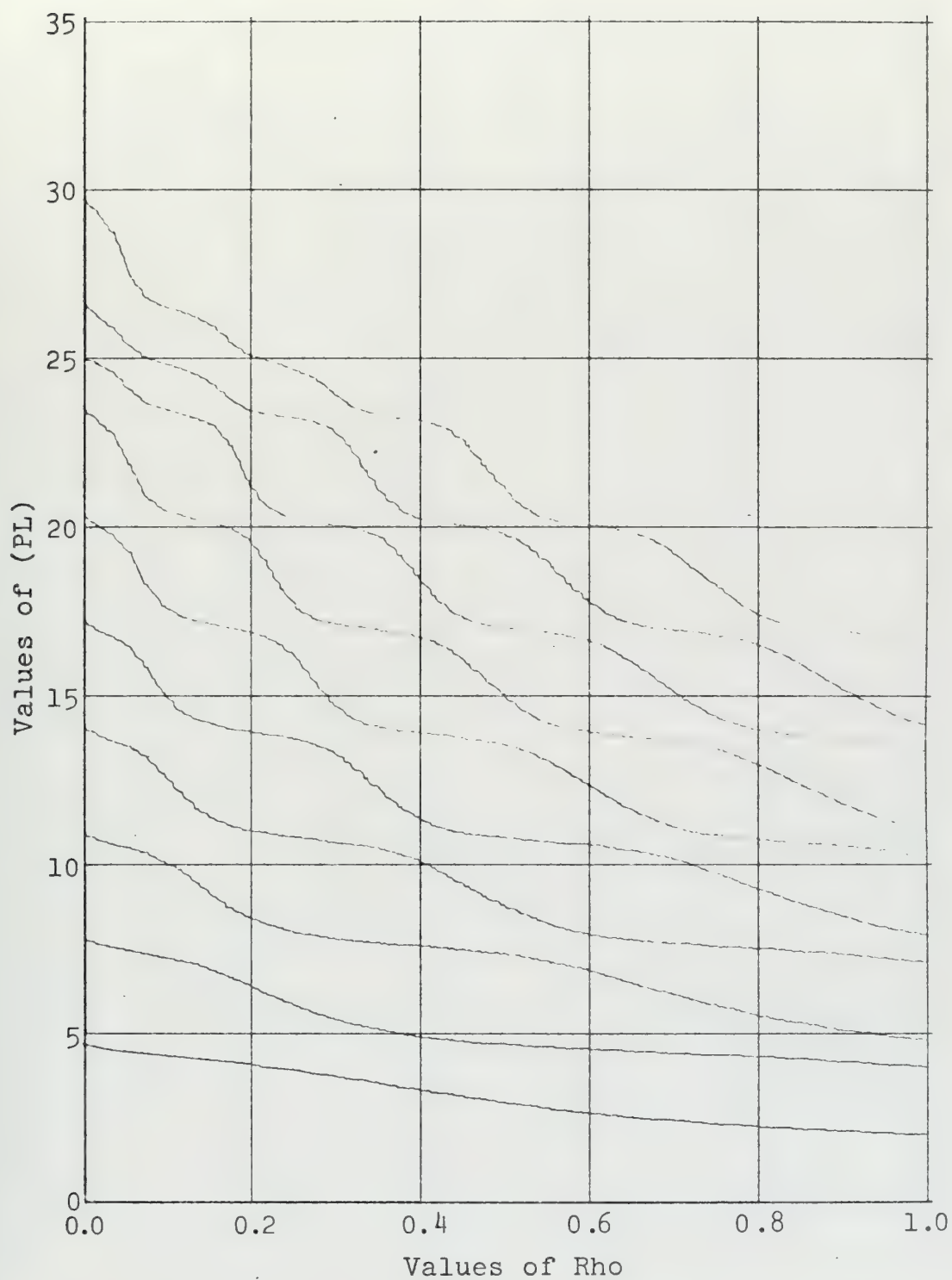


Figure 3-4. Out-of-plane Vibrations, $\beta = 0.005$.

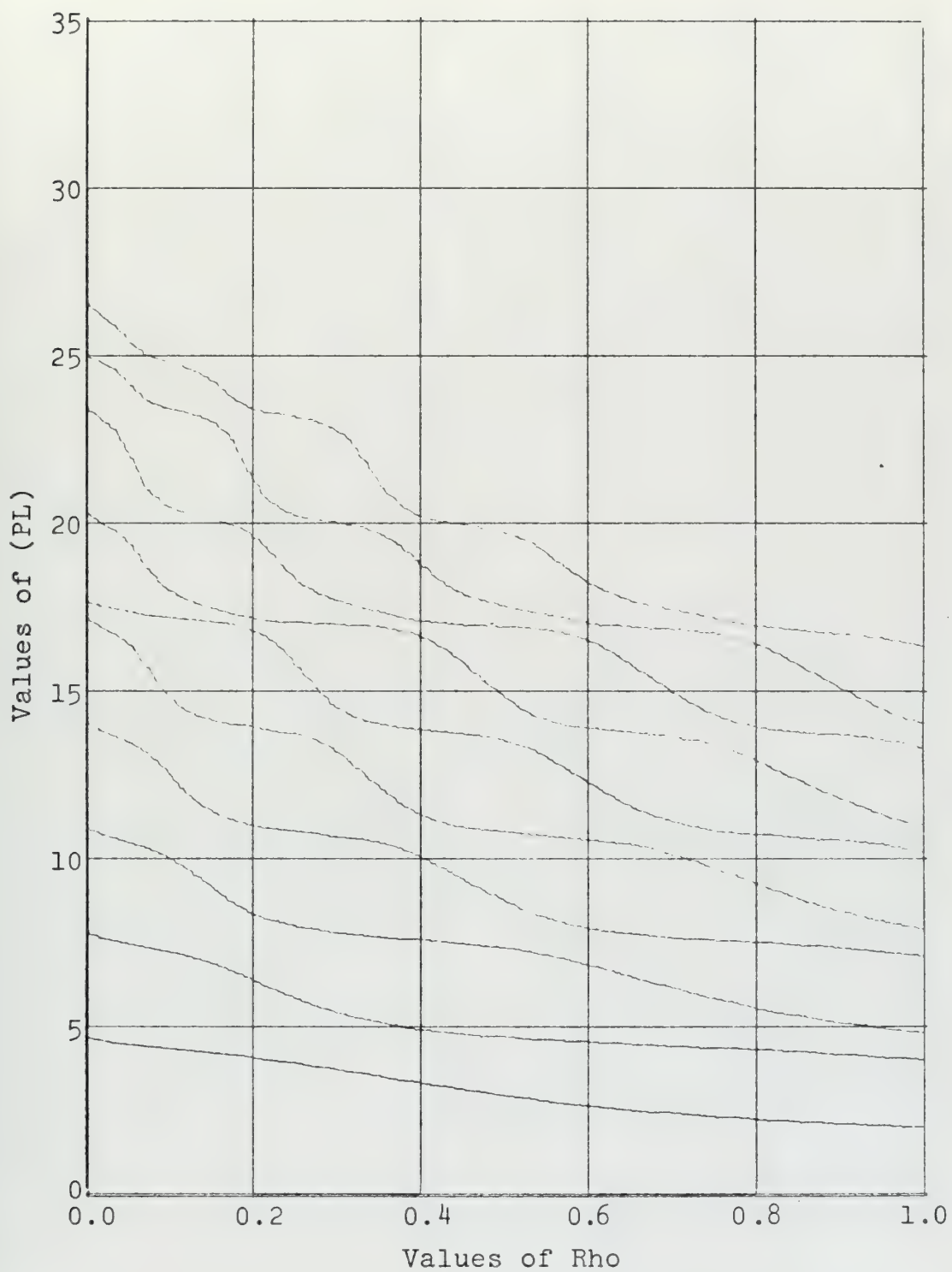


Figure 3-5. Out-of-plane Vibrations, $\beta = 0.01$.

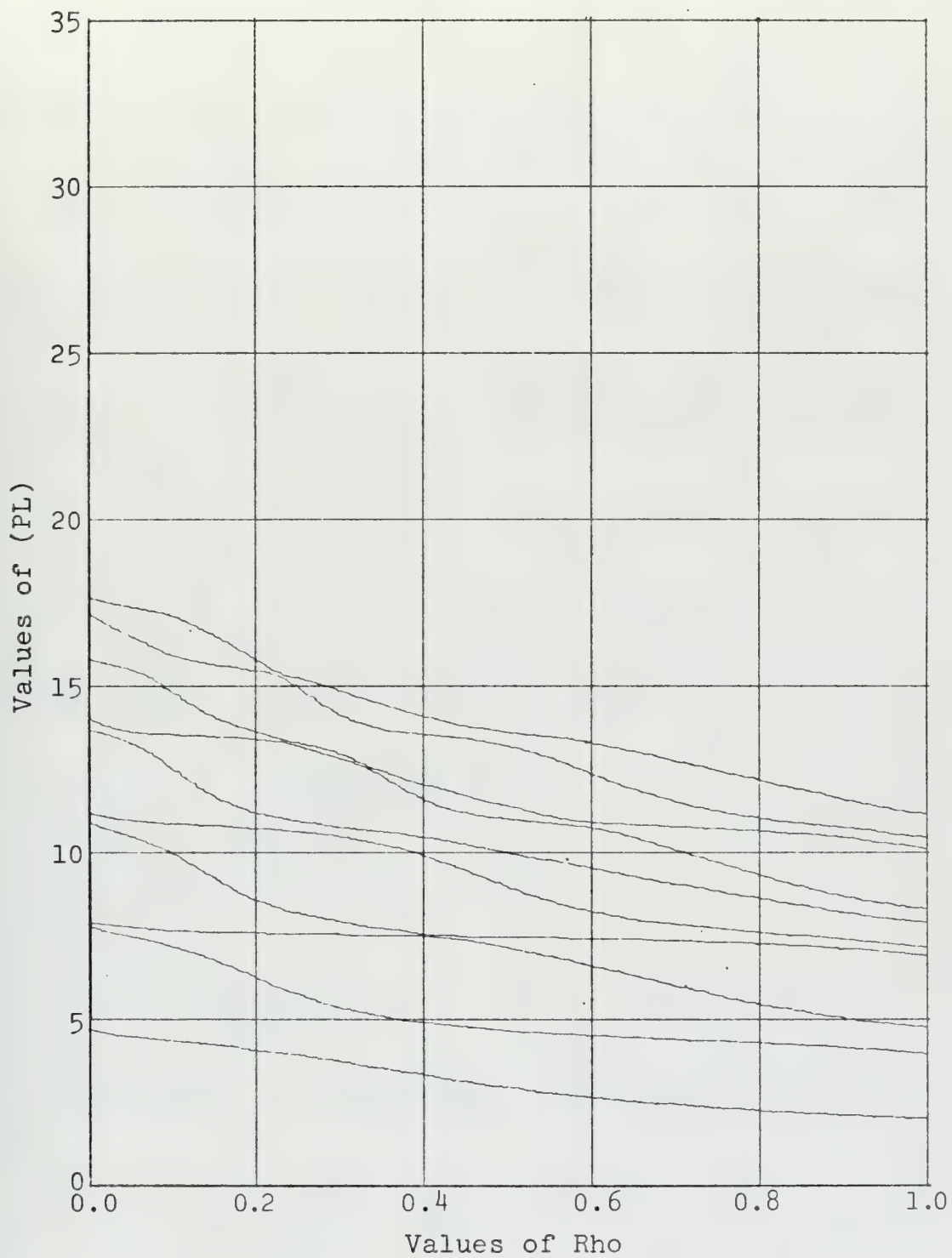


Figure 3-6. Out-of-plane Vibrations, $\beta = 0.05$.

$$\rho = \frac{L_2}{L_1} = 0.62373737\ldots$$

and β can be found from equation (2-29).

$$\beta = \sqrt{\frac{1.3v}{\mu L_1^2}} \quad (3-1)$$

where v is the total mass moment of inertia per unit length for the torsion and μ is the total mass per unit length undergoing lateral vibrations. Assuming that the contents (water) do not rotate, then

$$v = v_{\text{pipe}} + v_{\text{insulation}}$$

$$\mu_{\text{total}} = \mu_{\text{pipe}} + \mu_{\text{insulation}} + \mu_{\text{contents}}$$

numerical values of v_{total} and μ_{total} are

$$v_{\text{total}} = 0.0509089405 \text{ lb-sec}^2$$

$$\mu_{\text{total}} = 1.068875490 \text{ lb-sec}^2\text{-in}^{-2}$$

and from equation (3-1)

$$\beta = 0.0150806896$$

For evaluation of frequencies, equation (2-22) should be used as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L_1^4}}$$

then

$$\sqrt{\frac{EI}{\mu L_1^4}} = 8.221317961 \text{ sec}^{-1}$$

To solve the problem, values of $(PL)_1$ can be found from the graphs. But no graph is available for the value of β found above. Therefore interpolation is required between the values taken from Figure 3-5 and from Figure 3-6. Values of $(PL)_1$ are given in the table below where the exact solution was obtained by a computer solution using the specified value for β .

MODE NO.	EXACT SOLUTION	SOLUTION OBTAINED BY LINEAR INTERPOLATION
1	2.624579	2.624302
2	4.533513	4.521991
3	6.729044	6.706905
4	7.862686	7.817582
5	10.518163	10.230722
6	11.908733	11.698410
7	13.730277	13.442683
8	14.259158	15.619057
9	16.376500	16.379861
10	17.185014	17.403866

As seen from the table, for the lower modes, linear interpolation gives good results. However there is greater error in using linear interpolation for the higher frequencies. One can make what appears to be the reasonable conjecture that if one were to plot curves with (PL) as ordinate and β as abscissa and with different curve sheets corresponding to

fixed values of ρ , there might be some crossing of individual curves; that is, for certain values of β two distinct modes might have the same frequency. Clearly this would confuse the significance of characterizing different modes simply by ordinal number in the order of increasing frequency and would indicate that a more sophisticated interpolation is required. However, no such graphs were prepared and there do not seem to be enough values of β represented in the graphs at hand to permit making replots. Thus it is suggested that some future investigator might find it interesting to employ the computer programs, incorporating slight modifications, to obtain such graphs and thus investigate the reasons for inaccurate interpolation for the higher modes.

IV. ACCURACY AND ASSURANCE OF SOLUTION

The allowable error permitted in determining the roots is 10^{-6} and this was assumed to be satisfactory for practical purposes. The allowable error can be changed from 10^{-6} to any desired value by changing the value EPS in the computer programs. This can be done by changing the card 000090 in the program IN-PLANE and card 000100 in the program OUT-OF-PLANE.

Assurance of solution was established in the following ways.

1. In the case ρ is equal to zero, the ell-shaped configuration becomes a simple built-in beam and solution for this case can be compared with the classical solution.

Degenerate Case; $\rho = 0$			
Mode No.	Values of (PL)		
	Comparison Values*	In - plane Solution	Out-of-plane Solution
1	4.7300407	4.7300289	4.7300411
2	7.8532046	7.8531849	7.8532047
3	10.9956078	10.9955803	10.9956081
4	14.1371655	14.1371302	14.1371655
5	17.2787597	17.2787165	17.2787594
6	20.4203522	20.4203012	20.4203526
7	23.5619449	23.5618858	23.5619449
8	26.7035376	26.7034814	26.7035412
9	29.8451302	29.8453738	29.8451861
10	32.9867229	32.9890104	32.9866973

* Obtained by computer solution of the equation $\text{Cosh}(PL) \text{Cos}(PL) = 1$.

In equation (2-21) when ρ is set to zero the value of the determinant is zero regardless of the value of (PL) . Therefore ρ was taken to be 0.00001.

In out-of-plane case for the larger values of β , pure torsional modes appeared and this can be seen from Figures (3-5) and (3-6). The values of (PL) given in the above table were taken when β is equal to 0.001.

2. Consider the cases shown in Figure 4-1.

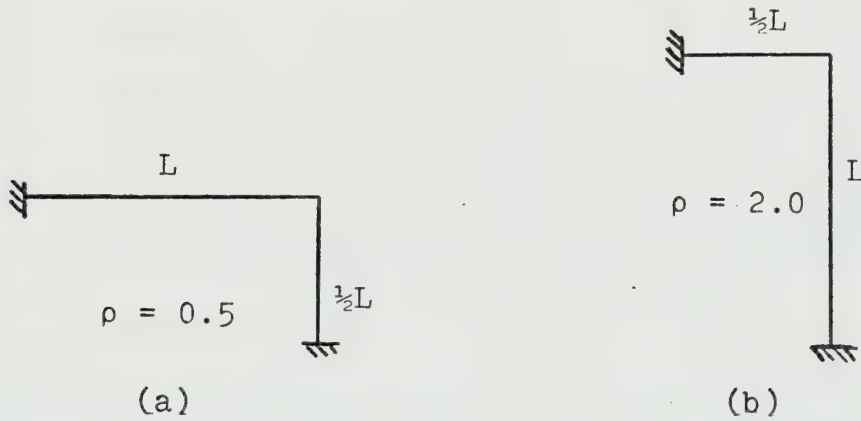


Figure 4-1. Cases Having Reciprocal Values of ρ .

The configurations shown in the above figure are identical, therefore two systems must have the same frequencies. Using equation (2-22) as

$$(PL)_{ai}^2 \sqrt{\frac{EI}{\mu L^4}} = (PL)_{bi}^2 \sqrt{\frac{EI}{\mu (\frac{L}{2})^4}}$$

then

$$\frac{(PL)_{ai}}{(PL)_{bi}} = 2$$

Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	4.3755336	2.1877668	2.000000000
2	7.2490132	3.6245066	2.000000000
3	8.7635958	4.3817979	2.000000000
4	10.707676	5.3538381	1.999999999
5	13.540341	6.7701705	2.000000001
6	15.024013	7.5120064	1.999999999
7	16.987974	8.4939872	2.000000000
8	19.8231918	9.9115959	2.000000000
9	21.3082077	10.6541038	2.000000001
10	23.2712829	11.6356415	2.000000001

For out-of-plane solution in the case $\beta = 0.001$

Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	2.9832348	1.4916187	1.999998209
2	4.7014996	2.3507534	1.999996916
3	7.3772076	3.6886158	1.999993485
4	8.8416393	4.4208958	1.999965571
5	10.8159008	5.4080044	1.999980037
6	13.6035633	6.8018597	1.999977062
7	15.0968311	7.5488148	1.999894233
8	17.0625606	8.5315098	1.999946210
9	19.8673419	9.9339245	1.999948965
10	21.3568783	10.6796268	1.999777575

3. In the case ρ is equal to one, it is expected that among the in-plane solutions will be many for which the moment at the joint vanishes. Thus, these cases also pertain to the clamped-pinned simple beam. Now two clamped-pinned simple beams of different lengths, ℓ and ℓ^* , may have matching frequencies in their spectra. That is, for example, the second frequency for one may equal the first frequency for the other. Specifically the following case was considered.

The third frequency for an equal-leg ell is equal to the second frequency of a clamped-pinned beam of length ℓ , namely

$$\omega_3 = (PL)_3^2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_3 = 7.06858275$$

The first frequency of an equal-leg ell of length ℓ^* is equal to the first frequency of clamped-pinned beam of length ℓ^* , namely

$$\omega_1 = (PL)_1^2 \sqrt{\frac{EI}{\mu \ell^{*4}}}$$

where

$$(PL)_1 = 3.92660231$$

The ratio ρ was determined so that then the frequencies are equal, thus

$$\rho = \frac{\ell^*}{\ell} = \frac{(PL)_1}{(PL)_3} = 0.55550065$$

Then performing an analysis for an ell-shaped configuration with the value of ρ , found above, the second frequency is indeed,

$$\omega_2 = (PL)_2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_2 = 7.068384686$$

This value shows close agreement with the value

$$(PL)_3 = 7.0685275$$

given above.

APPENDIX A

DESCRIPTION OF PROGRAMS

A. GENERAL REMARKS

Programs IN-PLANE and OUT-OF-PLANE are FORTRAN-IV language digital computer programs designed for use in the vibration analysis of an ell-shaped uniform filament with clamped ends.

Both programs employ double precision arithmetic. Subroutines are provided in binary form. Trigonometric and hyperbolic functions, available in the IBM-360/67 library, were used.

B. PROGRAM STRUCTURE

Each program consists of a four section main program and two subroutines. The function of each section of the main program and of each subroutine is given below.

1. Main program subsections and their functions.

INPUT	Controls read-in of data and allocates storage locations for arrays and subscripted variables. The allowable error for determination of roots is specified in this section.
-------	---

ITERATION AND EVALUATION OF ROOTS	In this section the roots of the determinant are located in an interval by searching the sign changes of the determinant and then the method of successive bisectioning is used to evaluate the roots. The maximum number of iterations was limited to three hundred and this was assumed sufficient. If the number of iterations becomes as large as three
--	---

hundred the process terminates and the approximate root is tagged to indicate a possible inaccurate result.

OUTPUT Controls output format.

GRAPH Controls the graph output and uses the DRAW subroutine provided in the IBM-360/67 library.

2. Subroutines and their functions.

BRODE In program IN-PLANE equation (2-21) is stored and in program OUT-OF-PLANE equation (2-30) is stored. Evaluates each element of the determinant and obtains the value of the determinant by calling subroutine DTERM.

DTERM This subroutine computes the determinant of a matrix of real numbers by Gauss' method of elimination with row pivoting.

C. PROGRAM NOMENCLATURE

A Elements of determinants.

B Checks sign of determinant and if there is a sign change moves control to the method of successive bisectioning.

BB Values of β in subroutine BRODE.

BBB Checks the acceptability of roots.

BETA Value of β .

CEM Single precision version of R0.

DET Value of determinant.

EPS Allowable error in roots.

F Value of determinant.

N Index of modes (Ordinal number).

P Values of (PL) used to search interval.

R3, R4 Values of determinant in the method of successive bisectioning.

R0 Values of ρ .

ROOT	Roots of determinant (PL).
RR	Corresponds to RO in subroutine BRODE.
SEV	Single precision version of ROOT.
X	Middle of interval.
XX1; XX2	Upper and lower limits of interval containing the root.
YY	Values of P, X, XX1 and XX2 in subroutine BRODE.

APPENDIX B

INSTRUCTIONS FOR USE OF PROGRAMS

A. DATA REQUIRED

1. Data required for program IN-PLANE
 - a. Number of ρ 's (NRO).
 - b. Number of modes sought (MODE).
 - c. Graph control (NGRAPH).
 - d. Values of ρ (RO).
 - e. Graph title (ITITLE).
2. Data required for program OUT-OF-PLANE
 - a. Number of ρ 's (NRO)
 - b. Number of β 's (NBETA).
 - c. Number of modes sought (MODE).
 - d. Graph control (NGRAPH).
 - e. Values of β (BETA).
 - f. Values of ρ (RO).
 - g. Graph title (ITITLE).

B. DATA DECK ASSEMBLY

1. Program IN-PLANE

The data deck is ordered as follows:

- a. Problem statement card.
 - b. Values of ρ card(s).
 - c. Graph title cards.
- ##### 2. Program OUT-OF-PLANE

The data deck is ordered as follows:

- a. Problem statement card.
- b. Values of β card(s).
- c. Values of ρ card(s).
- d. Graph title cards.

C. LIMITING THE NUMBER OF ITERATIONS

In the programs the maximum number of iterations was set to 300. If the number of iterations reaches 300 for any root, the current value is assumed as a solution but in the output it is marked as ITER. EXCEEDED 300. A user either accepts the root or changes the cards 000430 in program IN-PLANE and 000460 in program OUT-OF-PLANE by changing 300 to any desired value.

D. GRAPH

A user can get graphical output by option. DRAW subroutine, available in IBM-360/67 library was used and three CALL DRAW statements control first, intermediate and last curves to be plotted on one graph. DRAW subroutine requires single precision arithmetic. Therefore variables CEM and SEV are introduced to change the values of RO and ROOT from double precision to single precision.

E. FORMAT OF DATA

1. Program IN-PLANE

- a. Problem statement card.

The card has 3 fields under the control of following field specification.

Field names and the information each field conveys are given below in order.

Field names and content.

- (1) NRO Number of ρ 's
- (2) MODE Number of modes sought.
- (3) NGRAPH Code specifying whether there is
to be graph output.
(NGRAPH = 0 No graph output)
(NGRAPH = 1 Graph will be output)

b. Values of ρ card(s).

Field specification 7F10.0.

If there are more than 7 different values of ρ , additional card(s) will be required with the same field specification.

c. Graph title cards.

The title of graph is read as data with the field specification 6A8. For the title two cards are required. The first card corresponds to the first line of the title and the second card corresponds to the second line.

2. Program OUT-OF-PLANE

a. Problem statement card.

The card has 4 fields under the following field specification

Field names are given below in order.

- (1) NRO Number of ρ 's.
- (2) NBETA Number of β 's.
- (3) MODE Number of modes sought.
- (4) NGRAPH Same as in program IN-PLANE

b. Values of β cards.

Field specification 7F10.0 and field name is BETA.

c. Values of ρ card(s).

Same as in program IN-PLANE.

d. Graph title cards.

Same as in program IN-PLANE.

APPENDIX C

FLOW DIAGRAMS

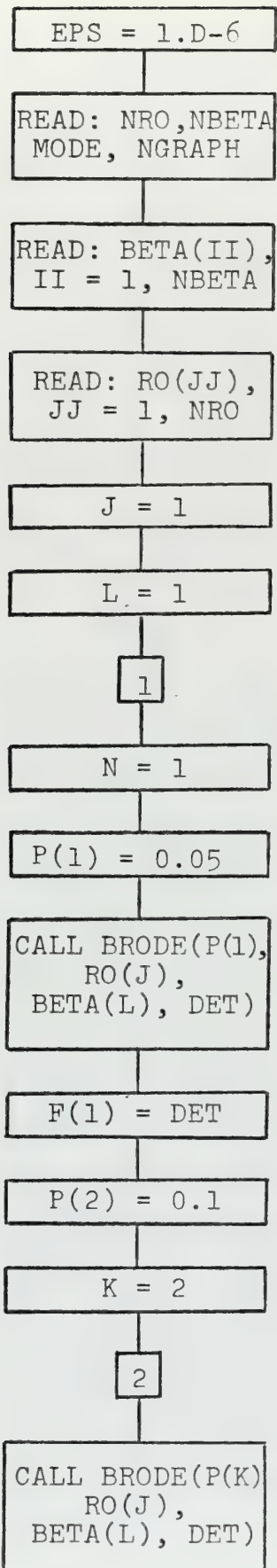
A. GENERAL

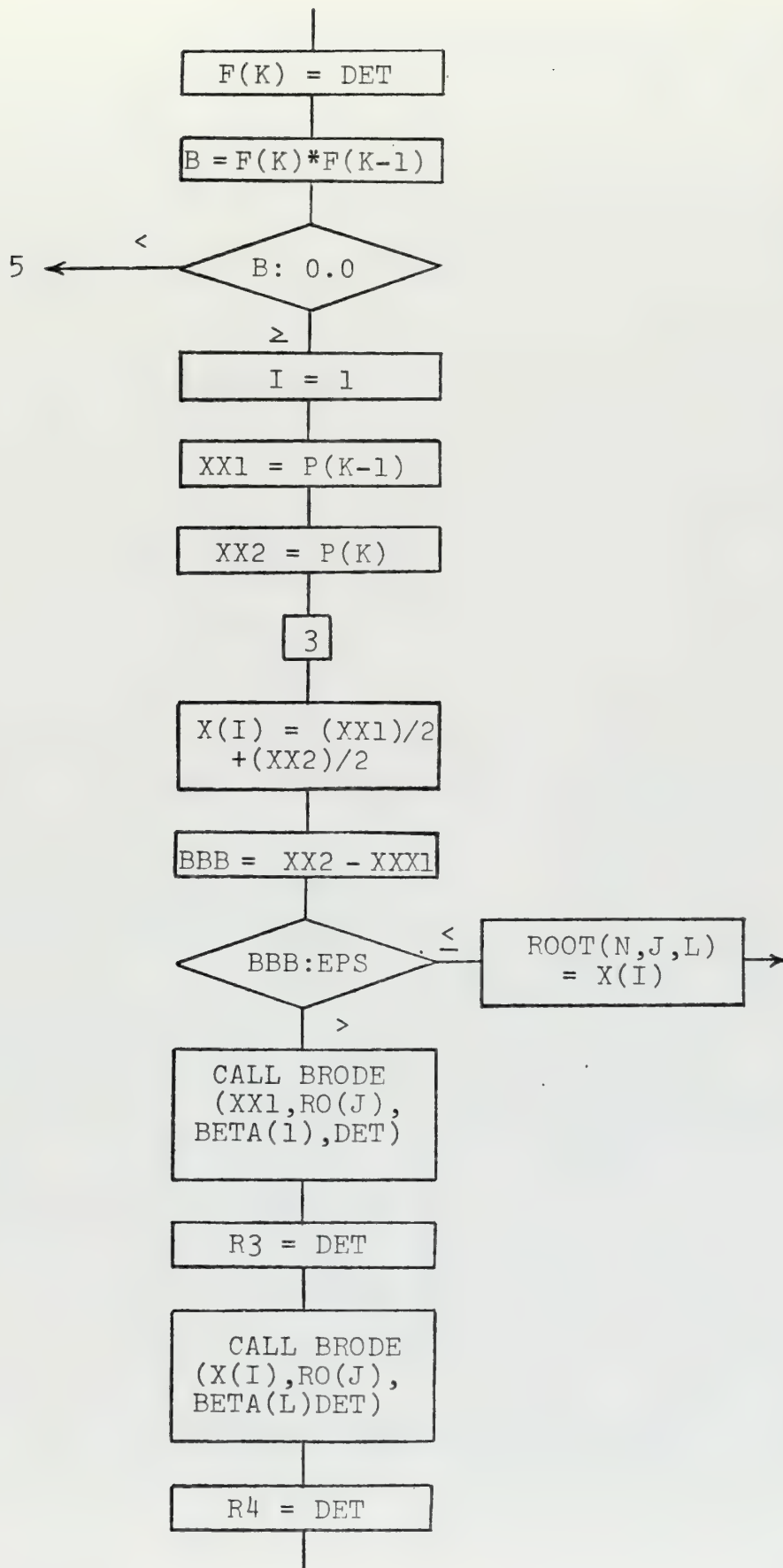
Flow diagram of the program OUT-OF-PLANE is given. This flow diagram includes all parts of the main program. Flow diagrams of subroutines BRODE and DTERM are not given and their functions were explained in Section A-2 in Appendix A.

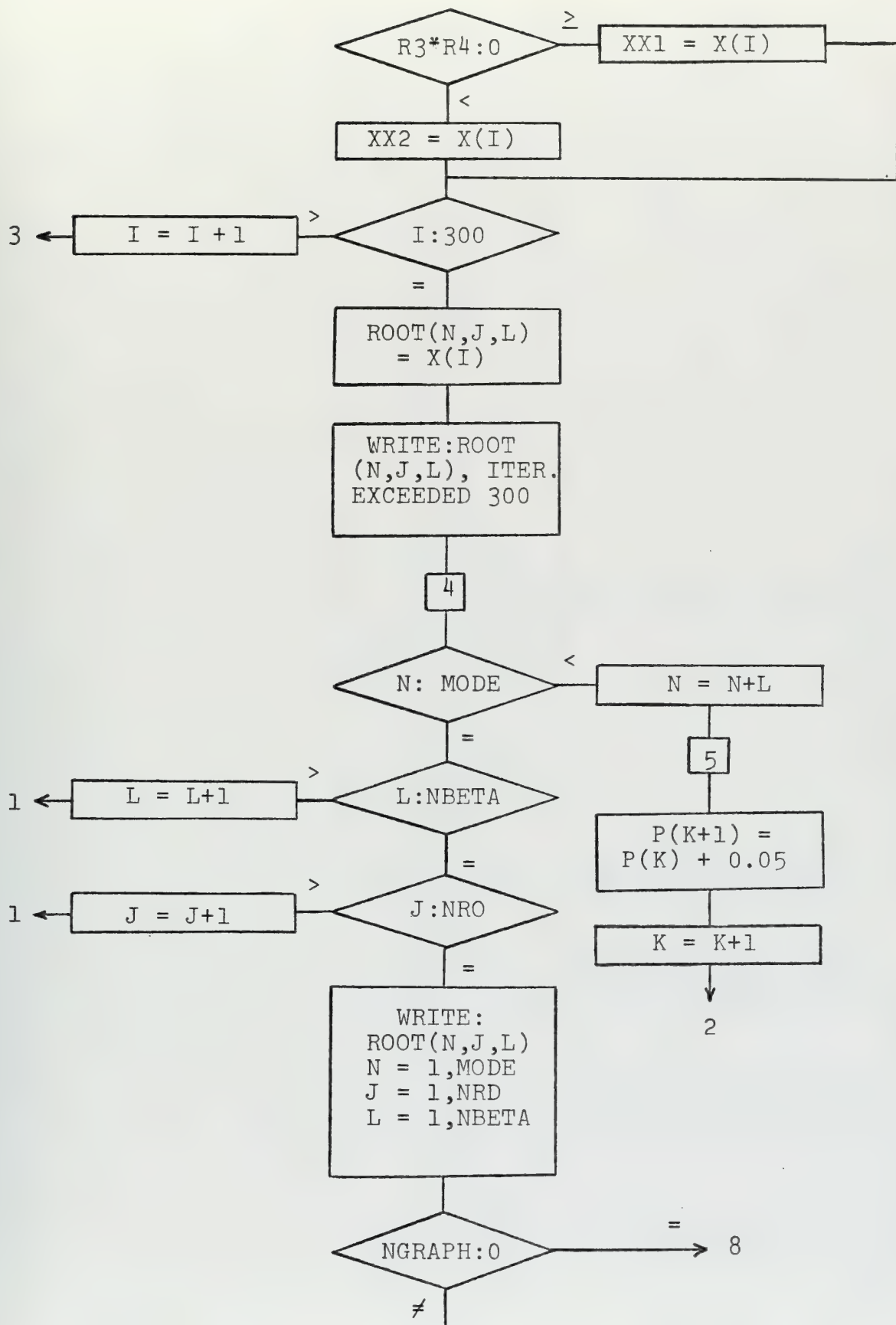
Program IN-PLANE is similar to OUT-OF-PLANE, therefore its flow diagram is not given.

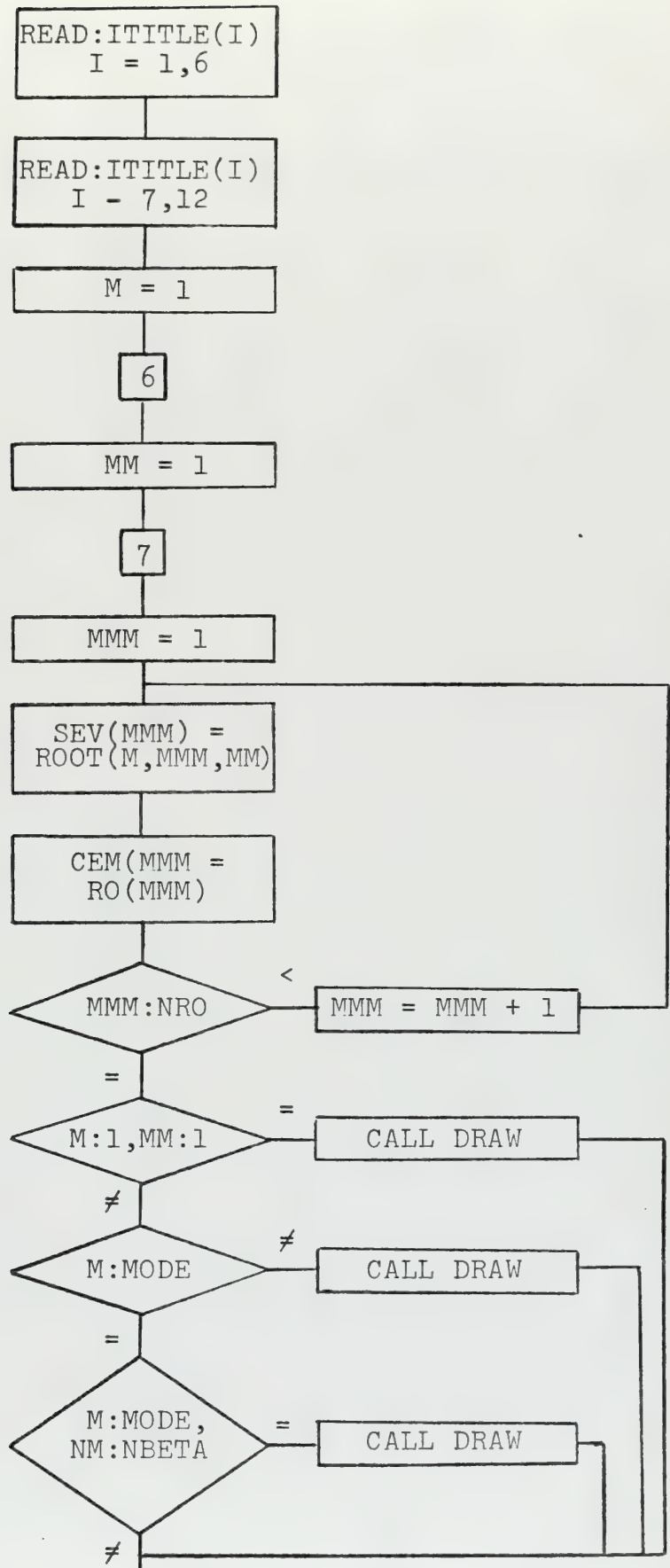
B. FLOW DIAGRAM OF PROGRAM OUT-OF-PLANE

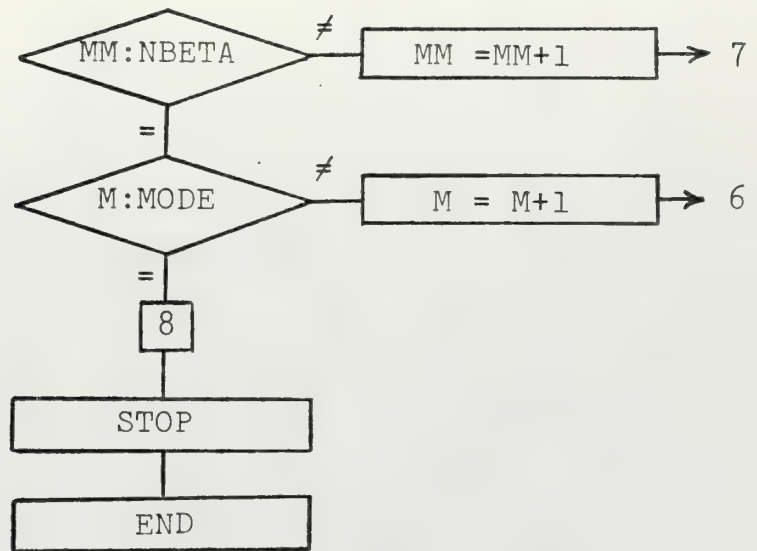
Flow diagram follows.











APPENDIX D

PROGRAM LISTING

A. INDEX

1. Program IN-PLANE

Main program:

<u>Section</u>	<u>Sequence number</u>	<u>Page</u>
INPUT	000090 - 000130	53
ITERATION AND EVALUATION OF ROOTS	000170 - 000550	53
OUTPUT	000590 - 000770	54
GRAPH	000810 - 001030	54

Subroutines:

BRODE	001060 - 001300	55
DTERM	001310 - 001610	55

2. Program OUT-OF-PLANE

Main program:

<u>Section</u>	<u>Sequence number</u>	<u>Page</u>
INPUT	000100 - 000150	57
ITERATION AND EVALUATION OF ROOTS	000190 - 000590	57
OUTPUT	000630 - 000830	58
GRAPH	000870 - 001110	58

Subroutines:

BRODE	001140 - 001600	59
DTERM	001610 - 001910	60

B. LISTING


```

GO TO 150
4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
1, LAST)
GO TO 150
4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
1, LAST)
150 WRITE(6,4400) LAST
4400 FORMAT(1H0,5X,'LAST=',I2)
800 CONTINUE
7000 STOP
END
000950
000960
000970
000980
000990
001000
001010
001020
001030
001040
001050

```

```

SUBROUTINE BRODE(YY,RR,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(4,4)
Y=0.5D0
Z=0.0D0
U=YY*RR
A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
A(1,2)=Z
A(1,3)=Y*(DSINH(YY)-DSIN(YY))
A(1,4)=Z
A(2,1)=Z
A(2,2)=Y*(DCOSH(U)-DCOS(U))
A(2,3)=Z
A(2,4)=Y*(DSINH(U)-DSIN(U))
A(3,1)=Y*(DSINH(YY)+DSIN(YY))
A(3,2)=-Y*(DSINH(U)+DSIN(U))
A(3,3)=A(1,1)
A(3,4)=-A(2,2)
A(4,1)=Y*(DCOSH(YY)+DCOS(YY))
A(4,2)=Y*(DCOSH(U)+DCOS(U))
A(4,3)=A(3,1)
A(4,4)=-A(3,2)
CALL DTERM(4,A,DET,4)
RETURN
END
001060
001070
001080
001090
001100
001110
001120
001130
001140
001150
001160
001170
001180
001190
001200
001210
001220
001230
001240
001250
001260
001270
001280
001290
001300

```

```

SUBROUTINE DTERM(N,A,D,M)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M,M)
DC=1.0D0
10 DO 34 L=1,N
KP=0
Z=0.0D0
DO 12 K=L,N
001310
001320
001330
001340
001350
001360
001370
001380

```



```

11 IF(Z-DABS(A(K,L)))11,12,12
12 Z=DABS(A(K,L))
13 KP=K
14 CONTINUE
15 IF(L-KP) 13,20,20
16 DO 14 J=L,N
17 Z=A(L,J)
18 A(L,J)=A(KP,J)
19 A(KP,J)=Z
20 DD=-DD
21 IF(L-N) 31,40,40
22 LP1=L+1
23 DO 34 K=LP1,N
24 IF(A(K,L)) 32,34,32
25 RATIO=A(K,L)/A(L,L)
26 DO 33 J=LP1,N
27 A(K,J)=A(K,J)-RATIO*A(L,J)
28 CONTINUE
29 DO 41 K=1,N
30 DD=DD*A(K,K)
31 D=DD
32 RETURN
33 END

```

```

001390
001400
001410
001420
001430
001440
001450
001460
001470
001480
001490
001500
001510
001520
001530
001540
001550
001560
001570
001580
001590
001600
001610

```



```

3100 CEM(MMM)=RO(MMM)
      CONTINUE
      IF(M.EQ.1.AND.MM.EQ.1) GO TO 4100
      IF(M.NE.MODE) GO TO 4200
      IF(M.EQ.MODE.AND.MM.EQ.NBETA) GO TO 4300
4100 CALL DRAW(NRO,CEM,SEV,1,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
      1, LAST)
      GO TO 4400
4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
      1, LAST)
      GO TO 4400
4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
      1, LAST)
4400 WRITE(6,4500) LAST
4500 FORMAT(1H0,5X,'LAST=',I2)
3200 CONTINUE
3300 CONTINUE
7000 STOP
      END
000950
000960
000970
000980
000990
001000
001010
001020
001030
001040
001050
001060
001070
001080
001090
001100
001110
001120
001130

```

```

SUBROUTINE BRODE(YY,RR,BB,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
Y=0.5D0
Z=0.0D0
U=YY*RR
W=BB*YY*2
V=W*RR
A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
A(1,2)=-Y*(DCOSH(U)-DCOS(U))
A(1,3)=Z
A(1,4)=Z
A(1,5)=Y*(DSINH(YY)-DSIN(YY))
A(1,6)=-Y*(DSINH(U)-DSIN(U))
A(2,1)=Z
A(2,2)=Y*(DSINH(U)+DSIN(U))
A(2,3)=-DSIN(W)*1.3D0/BB
A(2,4)=Z
A(2,5)=Z
A(2,6)=-A(1,2)
A(3,1)=Y*(DSINH(YY)+DSIN(YY))
A(3,2)=Z
A(3,3)=Z
A(3,4)=DSIN(V)*1.3D0/BB
A(3,5)=A(1,1)
A(3,6)=Z
A(4,1)=A(1,5)
001140
001150
001160
001170
001180
001190
001200
001210
001220
001230
001240
001250
001260
001270
001280
001290
001300
001310
001320
001330
001340
001350
001360
001370
001380
001390
001400

```



```

A(4,2)=-A(1,6)
A(4,3)=Z
A(4,4)=Z
A(4,5)=Y*(DCOSH(Y)+DCOS(Y))
A(4,6)=Y*(DCOSH(U)+DCOS(U))
A(5,1)=A(4,5)
A(5,2)=Z
A(5,3)=Z
A(5,4)=-YY*DCOS(V)
A(5,5)=A(3,1)
A(5,6)=Z
A(6,1)=Z
A(6,2)=A(4,6)
A(6,3)=YY*DCOS(W)
A(6,4)=Z
A(6,5)=Z
A(6,6)=A(2,2)
CALL DTERM(6,A,DET,6)
RETURN
END

```

```

001410
001420
001430
001440
001450
001460
001470
001480
001490
001500
001510
001520
001530
001540
001550
001560
001570
001580
001590
001600

```

```

SUBROUTINE DTERM(N,A,D,M)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M,M)
DD=1.0DO
DO 34 L=1,N
  KP=0
  Z=0.0DO
  DO 12 K=L,N
    IF(Z-DABS(A(K,L)))11,12,12
  11 Z=DABS(A(K,L))
  12 CONTINUE
  13 IF(L-KP) 13,20,20
  DO 14 J=L,N
    Z=A(L,J)
    A(L,J)=A(KP,J)
    A(KP,J)=Z
  14 DD=-DD
  20 IF(L-N) 31,40,40
  31 LP1=L+1
  DO 34 K=LP1,N
    IF(A(K,L)) 32,34,32
  32 RATIO=A(K,L)/A(L,L)
  DO 33 J=LP1,N
    33 A(K,J)=A(K,J)-RATIO*A(L,J)
  34 CONTINUE

```

```

001610
001620
001630
001640
001650
001660
001670
001680
001690
001700
001710
001720
001730
001740
001750
001760
001770
001780
001790
001800
001810
001820
001830
001840
001850
001860

```



```
40 DO 41 K = 1, N  
41 DD=DD*A(K, K)  
D=DD  
RETURN  
END
```

```
001870  
001880  
001890  
001900  
001910
```


APPENDIX E

PROGRAM VIPIPE

A. GENERAL

As a part of the present thesis investigation, attention was given to a digital computer program called VIPIPE which was originally written by Fink [1] and later modified and augmented by Kim [2]. This program provides solutions for in-plane and out-of-plane vibration of a single plane piping configuration having quite general geometry (except, of course, for its limitation to a single plane). It was desired to convert this program from a FORTRAN 63 program for use with CDC-1604 computer formerly employed at the Naval Postgraduate School, to a FORTRAN IV program for the IBM 360/67 presently in use, in order to provide an independent means of checking the results obtained as indicated in the body of this thesis and, also, because it would be useful to have a program of such generality available. For details of the theory of VIPIPE, refer to the theses written by Fink and Kim.

Because of the limitations on word length in IBM 360/67, the converted program, which was called VIPIPE-1, has been written to employ double precision arithmetic. Certain improvements have been made in the capability to produce graphical output.

The effort to make this conversion has been found to be much greater than was anticipated. It seems that CDC-1604 computer is a more "forgiving" machine than is the IBM 360/67. In checking out the new program certain bugs came to light which were actually present in the earlier version but which apparently did not cause trouble. However, they represented some syntactical errors that resulted in failure on the IBM 360/67. In some cases it was clear how to modify the program so as to eliminate the errors but in other cases the difficulty was so deep in the logic that a correction could not be made without acquiring as deep a knowledge of the program as was possessed by original writers, Fink and Kim.

Thus it cannot be asserted that VIPIPE-1 has been successfully converted in every fine detail and it is recommended that additional efforts be made to assure the overall integrity of the program and to assess its limitations. It should be noted, however, that no gross errors have been found in any results obtained by use of the new program; the only question is that of the degree of accuracy that it is possible to obtain.

Comparisons of accuracy are difficult to make. In the first place, the word-length is different in the new program, and, other things being equal, one would expect the new program to provide better accuracy, and to maintain its integrity in the face of round-off error beyond the point where the earlier programs "bombed out." Thus, agreement

between Kim and Fink and disagreement with VIPIPE-1 does not mean that the former are "correct" and the latter is in error. Furthermore, Kim reports having discovered and corrected some bugs which he found in Fink's version. Also, there is some difficulty in understanding the precise details of some of the problems treated by Fink and Kim.

With this preface which admonishes the reader not to make superficial inferences from comparisons, in Section C some comparisons between results obtained by Kim and results obtained using VIPIPE are presented.

B. DATA DECK ASSEMBLY

The order of data deck is

1. Graph title data cards.
2. Problem statement card.
3. Boundary condition data card(s).
4. Branch joining angle data card(s).
5. Component extensive properties data card(s).
6. Component intensive properties data card(s).

Field specification and format of graph title cards are given in Section E-1-c of Appendix B and information for the remaining cards are given in [1].

C. SAMPLE PROBLEMS

Properties of piping systems analyzed in this section are given in [1]. Here, only numerical values of frequencies will be given.

1. System 1 (in-plane)

Mode No.	Comparison* Values	VIPIPE** Delta m. (D)	VIPIPE-1*** Delta m.
1	75.09994540	75.09994544	75.09994535
2	207.01589137	207.01589036	207.01589099
3	405.83391916	405.83393183	405.83391910
4	670.86408383	670.86310856	670.86408365
5	1002.15520225	1002.19978814	1002.15520252
6	1399.70436269		1399.70434917
7	1863.51172599		1863.51198134

*Sources of comparison frequencies for system 1 and system 2 are given in Section D.

**Frequencies are given in [1].

*** Frequencies are found by VIPIPE-1.

2. System 2 (in-plane)

Mode No.	Comparison Values	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	11.80213586	11.80213389	11.80213432
2	73.96272297	73.96272167	73.96272087
3	207.09776595	207.09776685	207.09776559
4	405.82896584	405.82894132	405.82896568
5	670.86435913	670.86200817	670.86435889
6	1002.15518770	1002.26833752	1002.15518758
7	1399.70436229	1399.08849534	1399.70437163

3. System 4 (in-plane)

Mode No.	VIPIPE M-D m. (D)	VIPIPE-1 M-D m.
1	66.69934902	67.31642169
2	171.22352538	172.86857155
3	335.35007083	339.09491357
4	535.64529414	544.08929568
5	773.44358487	794.57343086

4. System 5 (in-plane)

Mode No.	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	116.89479180	118.95185774
2	304.88354398	302.91288556
3	566.43670976	519.19656352

Mode No.	VIPIPE P.M. m.-C (D)	VIPIPE-1 P.M. m. -C
1		118.95185779
2		302.91288592
3		519.19656389
4	826.88281929	794.49003694
5	1126.83797508	1126.27858174
6	1666.58878192	1464.59530724
7	2099.52844030	1874.97923215

D. SOURCES OF COMPARISON FREQUENCIES FOR SYSTEM 1 AND SYSTEM 2

The governing fourth order equation of a homogeneous single component straight system

$$y^{IV} = \frac{\mu\omega^4}{EI} y \quad (E-1)$$

was solved to give

$$y = a\cos Px + b \cosh Px + c \sin Px + d \sinh Px$$

Upon applying boundary conditions for system 1* ($y = y' = 0$ at $x = 0, x = L$) and simplifying, it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = 1 \quad (E-2)$$

Upon applying the boundary conditions for system 2 ($y = y' = 0$ at $x = 0, y'' = y''' = 0$ at $x = L$), it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = -1 \quad (E-3)$$

The roots of equation E-2 and E-3, from which the natural frequencies of corresponding systems may easily be found, were obtained through iteration using a digital computer.

*System 1 is clamped-clamped end beam
System 2 is clamped-free end beam

E. PROGRAM LISTING

1. Index

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SFIELO	013910 - 014180	101
MATMUL	014210 - 014330	101
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FINMAT	014650 - 014920	102
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STATEM	016320 - 016840	106
STATEB	016870 - 017610	107
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2. Listing

PROGRAM VIPIPE-1

INPUT

IMPLICIT REAL*8(A-H,O-Z)

REAL*4 CET,ZERGU

REAL*8 ITITLE(12)

REAL LABEL//ONE//

REAL LABEL1//ONE//

REAL LABEL2//TWO//

REAL LABEL3//THRE//

REAL LABEL4//FOUR//

REAL LABEL5//FIVE//

REAL LABEL6//SIX//

REAL LABEL7//SEVN//

REAL LABEL8//EIT//

REAL LABEL9//NINE//

REAL LABEL10//TEN//

REAL LABEL//LAST//

COMMON AJT(50),R(50),AIX(50),AIY(50),PHI(20),SVI(6,1),

1DT(50),D(50),DI(50),TL(50),Z(50),G(50),E(50),AMU(50),THETA(50),RHO

2(50),OMEGAM(20),DEL(600),OMEGA(600),A(6,6),B(6,6),U(6,6),CLX(50),C

3LY(50),CLZ(50),CTX(50),CTY(50),CTZ(50),V(6,3),VV(6,3),UU(6,3),SVE(

46,1),SV(22),SVIP(6,22),SVO(22),SVOIP(6,22),NNBR(50),BAMU(50),THETR(

520),P(20),YY(3,3,23),R3(3,3),XX(3,3),EDL(600),ZERGU(600),CET(600)

SEVIM=1.0D0

EEVIN=0.0D0

NM=0

JJ=1

READ(5,2001) (ITITLE(I),I=1,6)

READ(5,2001) (ITITLE(I),I=7,12)

FORMAT(6A8)

2001 READ(5,1500) NS,NBR,IOP,HM,AMYK,SD,RI,OMEGAG,OMEGAI,GDABC,BRC,

450 GRAPH,REM,PNM,HL

1500 FORMAT(3I2,4F3.0,2F8.0,4F2.0,2F4.0)

IF(NS) 600,600,601

601 NMBR=NBR+2

IF(IOP-1) 1501,1501,1506

1501 READ(5,1502) (SV(I),I=1,NMBR)

1502 FORMAT(2F3.0)

DO 1505 J=1,NMBR

IF(SV(J)) 1503,1503,1505

1503 READ(5,1504) (SVIP(I,J),I=1,6)

1504 FORMAT(6F2.0)

1505 CONTINUE

IF(IOP) 1506,1506,1523

1506 READ(5,1507) (SVO(I),I=1,NMBR)

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000200
000210
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000230
000240
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000270
000280
000290
000300
000310
000320


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1507 FORMAT(22F3.0)
DO 1510 J=1,NMBR
IF(SVOP(J)) 1508,1508,1510
1508 READ(5,1509) (SVOP(I,J),I=1,6)
1509 FORMAT(6F2.0)
1510 CONTINUE
1511 IF(NBR) 1513,1513,1511
1512 READ(5,1512) (PHI(I),I=1,NBR)
1513 FORMAT(10F7.3)
1(I),CTX(I),CTY(I),DI(I),RHO(I),TL(I),THETA(I),CLX(I),CLY(I),CLZ
1514 FORMAT(3F5.2,2F6.2,6F7.2,I2)
1515 READ(5,1515) AAMU,AE,AG,DD
1516 IF(DD) 1516,1516,1518
DO 1517 I=1,NS
AMU(I)=AAMU
E(I)=AE
G(I)=AG
1517 GO TO 1520
1518 AMU(1)=AAMU
E(1)=AE
G(1)=AG
1519 READ(5,1519) (AMU(I),E(I),G(I),I=2,NS)
1520 FORMAT(F7.3,2E10.2)
1121 IF(OMEGAI) 1121,1121,1521
1521 OMEGA(1)=0.01D0
OMEGA(I)=OMEGAI
AMYK=AMYK-SEVIM
OMEGAO=OMEGAI
FFAC=0.5D0
L=1
M=1
MN=0
AA=EEVIN
BB=EEVIN
CC=EEVIN
FF=EEVIN
GG=EEVIN
RR=EEVIN
KK=0
PP=EEVIN
KA=0
KKK=0
AAMU=EEVIN
EE=EEVIN
AL=EEVIN
AD=EEVIN

```


1071	SVE(J,1)=SVIP(J,NMBR)	001770
1081	JK=0	001780
1085	IF(NBR) 1000,1000,1085	001790
1086	DO 1087 N=2,NN	001800
1087	IF(SV(N)) 1087,1087,1086	001810
1088	CALL STAVEC(SV(N),N,SVIP)	001820
1089	CONTINUE	001830
1090	N=2	001840
	II=1	001850
	HH=EEVIN	001860
	DO 1001 I=1,NS	001870
	MAT=0	001880
	BC=EEVIN	001890
	P(I)=CLX(I)+CLY(I)+CTZ(I)	001900
	IF(P(I)) 37,38,37	001910
37	CALL HANGER(CLX(I),CLY(I),CTZ(I),U)	001920
	GO TO 62	001930
38	IF(AMK) 39,41,39	001940
39	IF(RHO(I)) 40,42,40	001950
40	IF(Z(I)) 43,45,43	001960
41	IF(RHO(I)) 40,48,40	001970
42	IF(Z(I)) 46,47,46	001980
43	CALL CFIELD(R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),E(I),A)	001990
44	CALL POINT(AMU(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)	002000
	CALL MATMUL(A,B,U)	002010
	MAT=1	002020
	GO TO 62	002030
45	CALL STIFCO(THETA(I),RHO(I),U)	002040
46	CALL DISTM(AMU(I),AIY(I),OMEGA(L),TL(I),R(I),D(I),DI(I),G(I),SD,	002050
	1RI,E(I),FFAC,U)	002060
	GO TO 62	002070
47	CALL RIGID(AMU(I),TL(I),OMEGA(L),AIY(I),RI,U)	002080
	GO TO 62	002090
48	IF(Z(I)) 49,47,49	002100
49	CALL SFIELD(DI(I),TL(I),R(I),Z(I),G(I),SD,D(I),E(I),FFAC,A)	002110
	GO TO 44	002120
62	IF(NBR(I)) 81,80,81	002130
80	IF(I-1) 63,79,63	002140
79	CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)	002150
	GO TO 63	002160
63	CALL FINMAT(U,UU,V,MAT,A,Z(I))	002170
	DO 64 J=1,6	002180
	DO 64 K=1,3	002190
64	UU(J,K)=V(J,K)	002200
	IF(PM) 88,1113,84	002210
1113	IF(FINK) 88,84,88	002220
84	IF(BC) 88,88,1111	002230
		002240

1111	IF(BRC) 88,88,89	002250
89	CALL BRCOR(R3,UU,XX,JK,VV,PM)	002260
	MMM=N-1	002270
	DO 90 J=1,3	002280
	DO 90 K=1,3	002290
90	YY(J,K,MMM)=XX(J,K)	002300
88	GO TO 1001	002310
81	IF(NNBR(I)-1) 83,83,65	002320
83	CALL STATEB(SVIP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)	002330
65	CALL FINBRA(U,VV,V,MAT,A,Z(I))	002340
	DO 66 J=1,6	002350
	DO 66 K=1,3	002360
66	VV(J,K)=V(J,K)	002370
	IF(NNBR(I)) 68,67,67	002380
67	IF(NNBR(I)-3) 1001,68,68	002390
68	CALL BRANCH(R3,VV,PHI(II),U)	002400
	N=N+1	002410
	II=II+1	002420
	MAT=0	002430
	BC=SEVIM	002440
	GO TO 63	002450
	CONTINUE	002460
1001	GO TO 2000	002470
		002480
		002490
		002500
		002510
		002520
		002530
		002540
		002550
		002560
		002570
		002580
		002590
		002600
		002610
		002620
		002630
		002640
		002650
		002660
		002670
		002680
		002690
		002700
		002710
		002720

1013	IF(SVO(1)) 1013,1023,1013
89	CALL STAVEO(SVO(1),1,SVI)
	GO TO 1043
1023	DO 1033 J=1,6
1033	SVI(J,1)=SVOP(J,1)
1043	IF(SVO(NMBR)) 1053,1063,1053
1053	CALL STAVEO(SVO(NMBR),1,SVE)
	GO TO 1083
1063	DO 1073 J=1,6
1073	SVE(J,1)=SVOP(J,NMBR)

998	P1=SEVIM
	P2=SEVIM
	D1=EEVIN
	D2=EEVIN
	Q1=SEVIM
	Q2=SEVIM
	Q3=SEVIM
	B1=EEVIN
	B2=EEVIN
	B3=EEVIN
	FINK=EEVIN

	CONTROL OUT OF PLANE
--	----------------------

C	
C	
C	
C	


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1083 JK=1
1093 IF(NBR) 1002,1002,1093
1095 DO 1095 N=2,NNN
1094 IF(SVO(N)) 1095,1095,1094
1095 CALL STAVED(SVO(N),N,SVOP)
1002 CONTINUE
N=2
II=1
HH=EEVIN
DO 1003 I=1,NS
BC=EEVIN
MAT=0
P(I)=CTX(I)+CLZ(I)+CTY(I)
IF(P(I)) 1,2,1
1 CALL HANGE(CTX(I),CLZ(I),CTY(I),U)
GO TO 14
2 IF(AMK) 3,11,3
3 IF(RHO(I)) 4,8,4
4 IF(Z(I)) 6,5,6
5 CALL STIFOO(THETA(I),RHO(I),U)
GO TO 14
6 CALL CFIELO(AJT(I),R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),FF
1AC,A)
7 CALL POINO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)
CALL MATMUL(A,B,U)
MAT=1
GO TO 14
8 IF(Z(I)) 10, 9,10
9 CALL RIGIO(AMU(I),TL(I),AIX(I),OMEGA(L),AIY(I),AJT(I),RI,U)
GO TO 14
10 CALL DISTMO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),AJT(I),R(I),D(I),
DI(I),G(I),SD,RI,FFAC,U)
GO TO 14
11 IF(RHO(I)) 4,12,4
12 IF(Z(I)) 13,9,13
13 CALL SFIELO(DI(I),TL(I),AJT(I),R(I),Z(I),G(I),SD,D(I),FFAC,A)
GO TO 7
14 IF(NNBR(I)) 201,200,201
200 IF(I-1) 15,202,15
202 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
GO TO 15
15 CALL FINMAT(U,UU,V,MAT,A,Z(I))
DO 16 J=1,6
DO 16 K=1,3
16 UU(J,K)=V(J,K)
1114 IF(PM) 207,1114,85
85 IF(FINK) 207,85,207
IF(BC) 207,207,1112

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1112 IF(BRC) 207,207,208
208 CALL BRCOR(R3,UU,XX,JK,VV,PM)
      MMM=N-1
      DO 209 J=1,3
      DO 209 K=1,3
      YY(J,K,MMM)=XX(J,K)
207 GO TO 1003
201 IF(NNBR(I)-1) 204,204,17
204 CALL STATEB(SVOP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
17 CALL FINBRA(U,VV,V,MAT,A,Z(I))
      DO 18 J=1,6
      DO 18 K=1,3
      VV(J,K)=V(J,K)
18 IF(NNBR(I)) 82,19,19
19 IF(NNBR(I)-3) 1003,82,82
82 CALL BRANCO(R3,VV,PHI(II),U)
      N=N+1
      II=II+1
      MAT=0
      BC=SEVIM
      GO TO 15
      CONTINUE

1003
C
C
C
2000 CALL DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,PM,D1,
102)
      IF(FINK) 87,87,86
87 IF(PM) 99,99,98
98 IF(KKK) 7030,7030,301
301 GO TO 6001
7030 IF(FF-SEVIM) 7051,7060,7051
7060 KK=1
      M=M-1
      GO TO 3000
7051 IF(HH-SEVIM) 300,6001,300
300 IF(SS) 303,302,302
303 DEL(L)=X2-Y2
      D1=X1
      D2=(X2+Y2)/2.0D0
      GO TO 61
302 DEL(L)=X1-Y1
      D1=(X1+Y1)/2.0D0
      D2=X2
      GO TO 61
99 DEL(L)=V(1,1)*V(2,2)*V(3,3)-V(1,3)*V(2,2)*V(3,1)+V(1,2)*V(2,3)*
1V(3,1)-V(1,2)*V(2,1)*V(3,3)+V(1,3)*V(2,1)-V(1,1)*V(3,2)*
2V(2,3)

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003220
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003250
003260
003270
003280
003290
003300
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003505
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003660
003670

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21	GO TO 61		003680
	PDEL=DEL(L)		003690
22	GO TO 25		003700
23	IF(AA) 23,23,24		003710
	PDEL=DEL(L-1)		003720
24	GO TO 25		003730
	PDEL=DEL(C		003740
25	IF(PDEL*DEL(L)) 26,26,27		003750
26	IF(AA) 28,28,57		003760
27	BB=EEVIN		003770
	IF(AA) 35,35,52		003780
28	AA=SEVIM		003790
	BB=SEVIM		003800
	DEL(C=DEL(L-1)		003810
	OMEGAC=OMEGA(L-1)		003820
29	GO TO 59		003830
	OMEGAM(M)=OMEGAC+(OMEGA(L)-OMEGAC)*DABS(DEL(C)/(DABS(DEL(C)+		003840
	1DABS(DEL(L)))		003850
30	GO TO 32		003860
	OMEGA(L+1)=OMEGA(L)-0.9D0*DOM		003870
31	L=L+1		003880
	DOM=0.5D0*DOM		003890
	BL=L		003900
7053	IF(HL-BL) 7053,7053,86		003910
	KA=1		003920
86	GO TO 3000		003930
32	IF(JK) 1000,1000,1002		003940
	BM=M		003950
33	IF(HM-BM) 3000,3000,33		003960
	M=M+1		003970
34	IF(M-3) 340,34,34		003980
340	ADOM=(OMEGAM(M-1)-OMEGAM(M-2))*0.1D0		003990
	AA=EEVIN		004000
	BB=EEVIN		004010
	CC=EEVIN		004020
	GG=EEVIN		004030
	DOM=ADOM		004040
	DEL(C=EEVIN		004060
	OMEGA(L+1)=OMEGAM(M-1)+2.0D0*CR		004070
	OMEGA(L)=OMEGA(L+1)		004080
	GO TO 31		004090
35	IF(OMEGA(L)-OMEGAC) 53,60,60		004100
50	OMEGA(L+1)=OMEGA(L)-(OMEGA(L)-OMEGAC)/2.0D0		004110
51	IF(OMEGA(L)-OMEGA(L+1)-CR) 51,51,31		004120
	CC=SEVIM		004130
52	GO TO 31		004140
	DEL(C=DEL(L)		004150
	OMEGAC=OMEGA(L)		004160


```

53 OMEGA(L+1)=OMEGA(L)+DOM
54 GO TO 31
55 IF(DEL C#DEL(L)) 29,29,55
56 OMEGA(M)=OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*DABS(DEL(L))/(DABS(DEL(L)
1)+DABS(DEL(L-1)))
57 GO TO 32
58 IF(OMEGA(L)-OMEGA(L-1)) 21,21,22
59 IF(BB) 58,58,50
60 BB=SEVIM
61 IF(OMEGA(L)-OMEGA(L-1)-CR) 29,29,30
62 M=M-1
63 GO TO 3000
64 IF(GG) 70,70,72
65 IF(DEL(L)) 77,71,77
66 GG=SEVIM
67 OMEGA(L+1)=OMEGA(L)-CR
68 GO TO 31
69 IF(GG-SEVIM) 73,73,74
70 GG=2.0DO
71 HDEL=DEL(L)
72 OMEGA(L+1)=OMEGA(L)+2.0DO*CR
73 GO TO 31
74 TDEL=DEL(L)*HDEL
75 IF(TDEL) 75,76,76
76 OMEGA(M)=OMEGA(L-2)
77 GO TO 33
78 KK=1
79 M=M-1
80 GO TO 3000
81 IF(CC) 56,56,54
82 OUTPUT
83 IF(NM) 400,400,403
84 WRITE(6,401)
85 FORMAT(1H1//)
86 WRITE(6,402)
87 FORMAT(16HPROGRAM VIPIPE-1,34X,15H M.CETIN DERIN,38X,14HDECEMBER
1 1970//,10X,108H
2 SYSTEM ARE DETERMINED BY AN ITERATIVE MODAL FREQUENCIES OF A PLANAR PIPING
3 THOD OF TRANSFER MATRICES.//119H *****
4 *****
5 *****//)
88 IF(NM)4058,4058,404
89 WRITE(6,405)
90 FORMAT(1H1//)
91 NM=1
92 IF(JK) 4060,4060,4065

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004170
004180
004190
004200
004210
004220
004230
004240
004250
004260
004270
004280
004290
004300
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CC



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4060 WRITE(6,4070) JJ
4070 FORMAT (50X,8HPROBLEM I2,/,35X,47HIN PLANE MODE FREQUENCIES (RADIA
1NS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /)
GO TO 4079
4065 WRITE(6,4075) JJ
4075 FORMAT(50X,8HPROBLEM I2,/,33X,51HOUT OF PLANE MODE FREQUENCIES (RA
1DIANS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /)
4079 MH=M
WRITE(6,408) (M,OMEGAM(M),M=1,MH)
408 FORMAT (46X,I2,17X,F25.16 /)
IF(KA) 5000,5000,5010
5010 WRITE(6,5020)
5020 FORMAT(10X,53HPROBLEM TERMINATED DUE TO OMEGA STORAGE LIMITATION.
1 /)
5000 IF(KK) 4080,4090,4080
4080 WRITE(6,4085)
4085 FORMAT(10X,59HPROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITAT
1ION OF COMPUTER. /)
4090 IF(AMK)412,409,412
409 WRITE(6,411)
411 FORMAT (16X,53H
1ND )
GO TO 414
412 WRITE(6,413)
413 FORMAT(24X,45HA DISTRIBUTED MASS APPROACH WAS EMPLOYED AND )
414 IF(SD)415,415,420
415 IF(RI)416,416,418
416 WRITE(6,417)
417 FORMAT (2X,67HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA W
1ERE CONSIDERED. /)
GO TO 4610
418 WRITE(6,419)
419 FORMAT (3X,63HTHE EFFECTS OF SHEAR DEFLECTION WERE CONSIDERED WHIL
1E THOSE OF /3X,35HROTATIONAL INERTIA WERE NEGLECTED. /)
GO TO 4610
420 IF(RI)421,421,423
421 WRITE(6,422)
422 FORMAT (3X,72HTHE EFFECTS OF ROTATIONAL INERTIA WERE CONSIDERED WH
1ILE THOSE OF SHEAR /3X,27HDEFLECTION WERE NEGLECTED. /)
GO TO 4610
423 WRITE(6,424)
424 FORMAT(3X,66HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WE
1RE NEGLECTED. /)
4610 IF(PM) 425,425,4713
4713 IF(SS) 4611,4612,4612
4611 IF(PP-SEVIM) 4613,4614,4615
4613 WRITE(6,4623)
4623 FORMAT(24X,35HPESTEL MAHREHOLTZ METHOD(A) USED. /)

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004990
005000
005010
005020
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005090
005100
005110
005120

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4614 GO TO 425
4624 WRITE(6,4624)
      FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(B) USED. /)
4615 GO TO 425
4625 WRITE(6,4625)
      FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(C) USED. /)
4612 GO TO 425
4616 IF(SS-SEVIM) 4616,4617,4618
4616 WRITE(6,4626)
4626 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(D) USED. /)
      GO TO 425
4617 WRITE(6,4627)
4627 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(E) USED. /)
      GO TO 425
4618 WRITE(6,4628)
4628 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(F) USED. /)
425 IF(PM)9010,9020,9000
9010 WRITE(6,9012)
9012 FORMAT(24X,20HDELTA METHOD USED. /)
      GO TO 9000
9020 WRITE(6,9022)
9022 FORMAT(24X,29HMODIFIED DELTA METHOD USED. /)
9000 WRITE(6,440)
440 *****
      IF(GDABC-SEVIM)4511,4520,4401
4401 WRITE(6,4402)
4402 FORMAT(35H SECTION LENGTH AND SUBSECTION DATA/23X,14HSECTION NUMBE
      IR,8X,25HLENGTH OF SECTION(INCHES),5X,21HNUMBER OF SECTIONS
      WRITE(6,443) (I,IL(I),Z(I),I=1,NS)
443 FORMAT (29X,13,23X,F7.2,21X,F4.0)
      LM=L
7055 IF(PM) 7054,7054,7055
441 WRITE(6,441)
      FORMAT (11H GRAPH DATA /22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
      IADIANS PER SECOND),3X,30H VALUE OF REMAINDER
      GO TO 7057
7054 WRITE(6,7056)
7056 FORMAT (11H GRAPH DATA / 22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
      IADIANS PER SECOND),3X,30HVALUE OF FREQUENCY DETERMINANT
7057 WRITE(6,444) (L,OMEGA(L),DEL(L),L=1,LM)
444 FORMAT (29X,13,23X,F11.6,18X,D15.8)
4520 WRITE(6,442)
442 FORMAT(1X,10HINPUT DATA /)
4521 WRITE(6,4521)
      FORMAT(1X,9HCOMPONENT,4X,8HDIAMETER,5X,14HWALL THICKNESS ,5X,6HLEN
      GTH,5X,9HRADIUS OF,9X,8HINCLUDED,5X,7HDENSITY,5X,7HELASTIC,5X,5HSH
      2EAR /57X,9HCURVATURE,5X,12HANGLE OF ARC,16X,7HMODULUS,5X,7HMODULUS
005130
005140
005150
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005490
005500
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005600

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3 /) DO 4525 I=1,NS
      IF(P(I))4524,4524,4523
4524 WRITE(6,4522) I,D(I),DT(I),TL(I),RHO(I),THETR(I),BAMU(I),E(I),G(I)
4522 FORMAT (3X,I3,8X,F7.3,10X,F6.3,7X,F8.3,4X,F8.3,9X,F6.2,8X,F7.2,4X,
      1E8.2,4X,E8.2)
      GO TO 4525
4523 SH=SEVIM
4525 CONTINUE
      IF(SH)395,395,4526
4526 WRITE(6,4532)
4532 FORMAT(/1X,7HHANGERS /)
      WRITE(6,4527)
4527 FORMAT (2X,9HCOMPONENT,14X,3HCLX,14X,3HCLY,14X,3HCLZ,14X,3HCTX,14X
      1,3HCTY,14X,3HCTZ /)
4528 DO 4530 I=1,NS
      IF(P(I))4530,4530,4529
4529 WRITE(6,4531) I,CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I),CTZ(I)
4531 FORMAT(3X,I3,15X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.
      12)
4530 CONTINUE
395 WRITE(6,448)
448 FORMAT (//20H BOUNDARY CONDITIONS )
      WRITE(6,449)((SVI(I,1),I=1,6)
      1)
449 FORMAT (5H SVI=6F3.0)
      WRITE(6,451)((SVE(I,1),I=1,6)
      1)
451 FORMAT (5H SVE=6F3.0)
      IF(NBR) 4511,4511,4502
4502 IF(JK)4505,4505,4515
4505 WRITE(6,4510)((SVIP(I,J),I=1,6),J=2,NNN)
4510 FORMAT (6F3.0)
      GO TO 4511
4515 WRITE(6,4517)((SVOP(I,J),I=1,6),J=2,NNN)
4517 FORMAT(6F3.0)
4511 CONTINUE
C SORT
      IF(GRAPH-SEVIM) 6001,6000,6001
6000 LM=LM-1
      DO 6100 L=1,LM
      EDL(L)=DEL(L)
      CONTINUE
6100 DO 6600 L=1,LM1
      LP1=L+1
      DO 6600 M=LP1,LM
      IF(OMEGA(L)-OMEGA(M)) 6600,6600,6003
6003 TEMP=OMEGA(L)
      OMEGA(L)=OMEGA(M)
      OMEGA(M)=TEMP

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006000
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006110
006120
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006140
006150
006160
006170
006180

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TEMP=EDL(L)
EDL(L)=EDL(M)
EDL(M)=TEMP
CONTINUE
C GRAPH
NUMPTS=60
LT1=LM/NUMPTS
IF(LT1) 6024,6025,6024
6025 NUMPTS=LM
L=1
DO 2015 J=1,60
ZERGU(J)=OMEGA(J)
CET(J)=EDL(J)
2015 CONTINUE
CALL DRAW(NUMPTS,ZERGU,CET,0,0,LABEL,ITITLE,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 6001
6024 DO 6020 J=1,17
LT2=0
LT2=LT2+J*NUMPTS
IF(LT2-LM)6020,6021,6022
6020 CONTINUE
6021 LT=LT1
GO TO 6023
6022 LT=LT1+1
NUMEND=LM-LT1*NUMPTS
6023 L=1
DO 7021 I=1,LT
NEVIN1=I-3
NEVIN2=I-5
NEVIN3=I-7
NEVIN4=I-9
NEVIN5=I-LT
IF(I-1) 6008,6009,6008
IF(I-LT) 8011,6011,6011
8011 IF(I-3) 6010,6010,8012
8012 IF(I-5) 6010,6010,8013
8013 IF(I-7) 6010,6010,8014
8014 IF(I-9) 6010,6010,8015
8015 IF(I-LT) 6010,6011,6011
6009 DO 6031 N=1,NUMPTS
EDL(N)=EDL(L+N-1)
CET(N)=EDL(N)
OMEGA(N)=OMEGA(L+N-1)
6031 ZERGU(N)=OMEGA(N)
WRITE(6,6027) NUMPTS,EDL(1)
6027 FORMAT(1H1,////5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)
CALL DRAW(NUMPTS,ZERGU,CET,1,0,LABEL1,ITITLE,0,0,0,0,0,9,10,1,

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1 LAST)
GO TO 7021
6010 L=L+NUMPTS
OMEGAX=OMEGA(L)
DO 6030 N=1,NUMPTS
EDL(N)=EDL(N+L-1)
CET(N)=EDL(N)
OMEGA(N)=OMEGA(L+N-1)
OMEGA(N)=OMEGA(N)-OMEGAX
ZERGU(N)=OMEGA(N)
6030 WRITE(6,6032) NUMPTS,EDL(1)
6032 FORMAT(///5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)
IF(NEVIN1.LT.0) GO TO 2002
IF(NEVIN1.EQ.0) GO TO 2003
IF(NEVIN2.LT.0) GO TO 2004
IF(NEVIN2.EQ.0) GO TO 2005
IF(NEVIN3.LT.0) GO TO 2006
IF(NEVIN3.EQ.0) GO TO 2007
IF(NEVIN4.LT.0) GO TO 2008
IF(NEVIN4.EQ.0) GO TO 2009
IF(NEVIN5.LT.0) GO TO 2010
2002 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL2,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2003 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL3,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2004 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL4,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2005 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL5,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2006 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL6,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2007 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL7,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2008 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL8,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2009 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL9,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2010 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL0,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021

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6011	L=L+NUMPTS	006970
	OMEGAX=OMEGA(L)	006980
	DO 6033 N=1,NUMEND	006990
	EDL(N)=EDL(N+L-1)	007000
	CET(N)=EDL(N)	007005
	OMEGA(N)=OMEGA(N+L-1)	007010
	OMEGA(N)=OMEGA(N)-OMEGAX	007020
6033	ZERGU(N)=OMEGA(N)	007025
	WRITE(6,6051) NUMEND,EDL(1)	007030
6051	FORMAT(/////5X,7HNUMEND=,I3,5X,7HEDL(1)=,E12.9)	007040
	CALL DRAW(NUMEND,ZERGU,CET,3,0,LABELL,ITITLE,0,0,0,0,0,0,9,10,1,	007050
	LAST)	007060
	GO TO 6001	007080
7021	CONTINUE	007090
6001	IF(HH-SEVIM) 8024,8023,8023	007100
8023	MH=M	007110
	LM=L	
8024	DO 446 M=1,MH	007130
446	OMEGAM(M)=EEVIN	007140
	DO 447 L=1,LM	007150
	OMEGA(L)=EEVIN	007160
	EDL(L)=EEVIN	007170
447	DEL(L)=EEVIN	007180
	DELC=EEVIN	007190
	OMEGAC=EEVIN	007200
	IF(HH-SEVIM) 8001,8022,8001	007210
8001	IF(PM) 8031,8033	007220
8033	IF(REM)8031,8032,8032	007230
8031	IF(MN)8022,8022,452	007240
8032	IF(REM-SEVIM) 8041,8042,8043	007250
8041	IF(PP-SEVIM) 8043,8043,8072	007260
8072	IF(SS) 8073,8074,8074	007270
8073	SS=EEVIN	007280
	GO TO 8022	007290
8074	IF(SS-SEVIM) 8042,8042,8075	007300
8075	SS=-SEVIM	007310
	PP=EEVIN	007320
	REM=-SEVIM	007330
	GO TO 8033	007340
8042	SS=SS-SEVIM	007350
	IF(SS)8051,8052,8053	007360
8051	SS=SEVIM	007370
	GO TO 8022	007380
8052	SS=2.0D0	007390
	GO TO 8022	007400
8053	SS=-SEVIM	007410
	REM=-SEVIM	007420
	GO TO 8033	007430

8043	PP=PP-SEVIM			007440
8061	IF(PP) 8061,8062,8063			007450
8062	PP=SEVIM			007460
8063	GO TO 8022			007470
452	PP=2.0D0			007480
8022	GO TO 8022			007490
	PP=EEVIN			007500
	REM=-SEVIM			007510
	GO TO 8033			007520
	PP=EEVIN			007530
	RR=EEVIN			007540
	SS=-SEVIM			007550
	L=1			007560
	M=1			007570
	AA=EEVIN			007580
	BB=EEVIN			007590
	CC=EEVIN			007600
	FF=EEVIN			007610
	GG=EEVIN			007620
	KK=0			007630
	KA=0			007640
	KKK=0			007650
	ADOM=BDOM			007660
	CR=BR			007670
	OMEGA(1)=OMEGAI			007680
	OMEGAO=OMEGAI			007690
	DOM=ADOM			007700
7040	IF(HH-SEVIM) 8021,7040,8021			007710
8191	IF(SS) 8191,8192,8192			007720
8192	IF(PP-SEVIM) 8091,8092,8093			007730
8091	IF(SS-SEVIM) 8291,8292,8293			007740
8081	WRITE(6,8081)			007750
	FORMAT(IH1,///18HMETHOD A	FAILED. ///		007760
	GO TO 8021			007770
8092	WRITE(6,8082)			007780
8082	FORMAT(IH1,///18HMETHOD B	FAILED. ///		007790
	GO TO 8021			007800
8093	WRITE(6,8084)			007810
8084	FORMAT(IH1,///18HMETHOD C	FAILED. ///		007820
	GO TO 8021			007830
8291	WRITE(6,8281)			007840
8281	FORMAT(IH1,///18HMETHOD D	FAILED. ///		007850
	GO TO 8021			007860
8292	WRITE(6,8282)			007870
8282	FORMAT(IH1,///18HMETHOD E	FAILED. ///		007880
	GO TO 8021			007890
8293	WRITE(6,8283)			007900
8283	FORMAT(IH1,///18HMETHOD F	FAILED. ///		007910


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8021 IF(PM) 453,453,8035
8035 IF(REM) 8037,8036,8036
8036 IF(JK) 998,999,998
8037 REM=ERM
453 IF(PM) 6081,6082,6080
6081 IF(PMM-SEVIM) 6084,6080,6084
6084 IF(PMM*0.1D0-10.1D0) 6086,6085,6086
6085 PM=SEVIM
GO TO 8036
6086 PM=EEVIN
GO TO 8036
6082 IF(PMM*0.1D0-1.1D0) 6080,6080,6085
6080 IF(MN) 9001,9001,8034
8034 MN=0
PM=PPM
GO TO 998
9001 JJ=JJ+1
DO 454 I=1,NS
454 P(I)=EEVIN
Z(I)=EEVIN
GO TO 450
600 STOP
END

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007960
007970
007980
007990
008000
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008070
008080
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008100
008110
008120
008130
008135

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SUBROUTINE SUBSEC (RHO,THETA,D,TL,HM,Z,OMEGAI,HMH)
IMPLICIT REAL*8(A-H,O-Z)
RHOD=DABS(RHO)
IF(HMH) 41,40,41
40 IF(HM-5.0D0) 8,8,7
8 IF(OMEGAI-800.0D0) 10,7,7
7 HM=6.0D0
GO TO 10
41 HM=HMH
10 RA=TL/D
3 IF(RHOD-0.0D0) 3,3,4
5 IF(RA-1.0D0) 5,5,6
5 Z=0.0D0
RETURN
6 IF(RA-3.0D0) 15,15,16
15 Z=2.0D0
RETURN
16 IF(RA-6.0D0) 17,17,18
17 Z=3.0D0
RETURN
18 IF(HM-3.0D0) 1,1,2
1 Z=6.0D0
RETURN

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008140
008150
008160
008170
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008640

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2 Z=HM*2.0D0
  RETURN
4 RD=RHOD/D
  IF(RD-1.0D0) 19,19,23
19 Z=0.0D0
  RETURN
23 IF(RD-3.0D0) 24,24,25
24 IF(THETA-0.44D0) 26,26,27
26 Z=0.0D0
  RETURN
27 IF(THETA-1.57D0) 28,28,29
28 Z=4.0D0
  RETURN
29 Z=6.0D0
  RETURN
30 IF(RD-6.0D0) 30,30,31
31 IF(THETA-0.44D0) 32,32,33
32 Z=4.0D0
  RETURN
33 IF(THETA-1.57D0) 34,34,35
34 Z=6.0D0
  RETURN
35 Z=8.0D0
  RETURN
36 IF(THETA-0.44D0) 36,36,3
  RETURN
  END

```

SUBROUTINE DISTM (AMU,AIY,OMEGA,TL,R,D,DI,G,SD,RI,E,FFAC,U)

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008680
008690
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008710
008720
008730
008740
008750
008760
008770
008780
008790
008800
008810
008820
008830
008840

```

1  GO TO 3
2  T=0.0D0
3  IF(SD) 4,4,5
4  S={AMU*(OMEGA*TL)**2}/(G*AREA)
  GO TO 6
5  S=0.0D0
6  SPT=S+T
  VV=DSQRT(P4+(S-T)**2/4.0D0)
  AL1=DSQRT(VV-SPT*0.5D0)

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AL2=DSQRT(VV+SPT*0.5D0)
CL=1.0D0/(AL1**2+AL2**2)
CHL1=((DEXP(AL1))+1.0D0/DEXP(AL1))/2.0D0
SHL1=((DEXP(AL1))-1.0D0/DEXP(AL1))/2.0D0
CL2=DCOS(AL2)
SL2=DSIN(AL2)
SBE=DSIN(BE)
CBE=DCOS(BE)
C00=CL*(AL2**2*CHL1+AL1**2*CL2)
C01=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)
C02=CL*(CHL1-CL2)
C03=CL*(SHL1/AL1-SL2/AL2)
DO 7 J=1,6
DO 7 K=1,6
U(J,K)=0.0D0
U(1,1)=CBE
U(1,6)=TL*SBE/(BE*ARA*E)
U(2,2)=C00-S*C02
U(2,3)=TL*(C01-SPT*C03)
U(2,4)=R*C02*TL**2
U(2,5)=(R*TL**3/P4)*((P4+S**2)*C03-S*C01)
U(3,2)=P4*C03/TL
U(3,3)=C00-T*C02
U(3,4)=TL*R*(C01-T*C03)
U(3,5)=U(2,4)
RR=1.0D0/R
U(4,2)=P4*RR*C02/TL**2
U(4,3)=RR*((P4+T**2)*C03-T*C01)/TL
U(4,4)=U(3,3)
U(4,5)=U(2,3)
U(5,2)=P4*RR*(C01-S*C03)/TL**3
U(5,3)=U(4,2)
U(5,4)=U(3,2)
U(5,5)=U(2,2)
U(6,1)=-AMU*TL*OMEGA**2*SBE/BE
U(6,6)=CBE
RETURN
END

```

```

SUBROUTINE DISTMO (AMU,AIX,AIY,OMEGA,TL,AJT,R,D,DI,G,SD,RI,FFAC,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
W=1.0D0/(G*AJT)
AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
BE=+DSQRT(AMU*(TL*AIX*OMEGA)**2**W)
P4=R*AMU*TL**4*OMEGA**2
IF(RI) 2,2,3

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009330
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2 T=R*AMU*(AIY*OMEGA*TL)**2
3 GO TO 4
4 T=0.0D0
5 IF(SD) 5,5,6
6 S=(AMU*(OMEGA*TL)**2)/(G*AREA)
7 GO TO 7
8 S=0.0D0
9 VV=+DSQRT(P4+(S-T)**2/4.0D0)
10 SPT=S+T
11 AL1=+DSQRT(VV-SPT*0.5D0)
12 AL2=+DSQRT(VV+SPT*0.5D0)
13 CL=1.0D0/(AL1**2+AL2**2)
14 CHL1=(DEXP(AL1)+1.0D0/DEXP(AL1))/2.0D0
15 SHL1=(DEXP(AL1)-1.0D0/DEXP(AL1))/2.0D0
16 CL2=DCOS(AL2)
17 SL2=DSIN(AL2)
18 SBE=DSIN(BE)
19 CBE=DCOS(BE)
20 CO0=CL*(AL2**2*CHL1+AL1**2*CL2)
21 CO1=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)
22 CO2=CL*((CHL1-CL2)
23 CO3=CL*((SHL1/AL1-SL2/AL2)
24 DO 1 J=1,6
25 DO 1 K=1,6
26 U(J,K)=0.0D0
27 U(1,1)=CBE
28 U(1,2)=TL*W*SBE/BE
29 U(2,1)=-AMU*TL*AIX**2*SBE/BE
30 U(2,2)=CBE
31 U(3,3)=CO0-S*CO2
32 U(3,4)=TL*(CO1-SPT*CO3)
33 U(3,5)=R*CO2*TL**2
34 U(3,6)=(R*TL**3/P4)*(-S*CO1+(P4+S**2)*CO3)
35 U(4,3)=P4*CO3/TL
36 U(4,4)=CO0-T*CO2
37 U(4,5)=TL*R*(CO1-T*CO3)
38 U(4,6)=U(3,5)
39 RR=1.0D0/R
40 U(5,3)=P4*RR*CO2/TL**2
41 U(5,4)=RR*(-T*CO1+(P4+T**2)*CO3)/TL
42 U(5,5)=U(4,4)
43 U(5,6)=U(3,4)
44 U(6,3)=P4*RR*(CO1-S*CO3)/TL**3
45 U(6,4)=U(5,3)
46 U(6,5)=U(4,3)
47 U(6,6)=U(3,3)
48 RETURN
49 END

```



```

SUBROUTINE RIGID (AMU,TL,OMEGA,AIY,RI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
AM=AMU*TL
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=1.0D0
U(6,1)=-AM*OMEGA**2
U(6,6)=1.0D0
U(2,2)=1.0D0
U(2,3)=TL
U(3,3)=1.0D0
U(4,2)=AM*TL*OMEGA**2/2.0D0
IF(RI) 2,2,3
2 U(4,3)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
GO TO 4
3 U(4,3)=AM*(OMEGA*TL)**2/6.0D0
4 U(4,4)=1.0D0
U(4,5)=TL
U(5,2)=-U(6,1)
U(5,3)=U(4,2)
U(5,5)=1.0D0
RETURN
END

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009652
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009660
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009675
009676

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SUBROUTINE RIGIO (AMU,TL,AIX,OMEGA,AIY,AJT,SDRI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
DO 2 J=1,6
DO 2 K=1,6
2 U(J,K)=0.0D0
AM=AMU*TL
U(1,1)=1.0D0
U(2,1)=-AMU*TL*(AIX*OMEGA)**2
U(2,2)=1.0D0
U(3,3)=1.0D0
U(3,4)=TL
U(4,4)=1.0D0
U(5,3)=AM*TL*OMEGA**2/2.0D0
IF(SDRI-O.0D0) 30,30,31
30 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
GO TO 1
31 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0)
1 U(5,5)=1.0D0

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009679
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009682
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009685
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009697

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009700
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U(5,6)=TL
U(6,3)=AM*OMEGA**2
U(6,4)=U(5,3)
U(6,6)=1.0D0
RETURN
END

009760
009762
009764
009770
009780
009790
009800
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009890
009900
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009940
009950
009960
009970

SUBROUTINE STIFCO (THETA,RHO,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
IF(RHO) 2,2,3
2 V=-1.0D0
GO TO 4
3 V=1.0D0
CT=DCOS(THETA)
ST=DSIN(THETA)
U(1,1)=CT
U(1,2)=-V*ST
U(2,1)=V*ST
U(2,2)=CT
U(3,3)=1.0D0
U(4,4)=1.0D0
U(5,5)=CT
U(5,6)=-V*ST
U(6,5)=V*ST
U(6,6)=CT
RETURN
END

010000
010010
010020
010030
010040
010050
010060
010070
010080
010090
010100
010110
010120
010130

SUBROUTINE STIFOO (THETA,RHO,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
IF(RHO) 1,1,2
1 V=-1.0D0
GO TO 3
2 V=1.0D0
CT=DCOS(THETA)
ST=DSIN(THETA)
DO 4 J=1,6
DO 4 K=1,6
4 U(J,K)=0.0D0
U(1,1)=CT
U(1,4)=ST*V

010140
010150
010160
010170
010180
010190
010200
010210
010220
010230

U(2,2)=CT
U(2,5)=ST*V
U(3,3)=1.0D0
U(4,1)=-ST*V
U(4,4)=CT
U(5,2)=-ST*V
U(5,5)=CT
U(6,6)=1.0D0
RETURN
END

010260
010270
010280
010290
010300
010310
010320
010330
010340
010350
010360
010370
010380
010390
010400
010410
010420
010430
010440
010450
010460
010470
010480
010490
010500
010510
010520
010530
010540
010550

SUBROUTINE STAVEC(SV,N,BC)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION BC(6,22)
DO 10 J=1,6
10 BC(J,N)=0.0D0
1 IF(SV-1.0D0) 1,1,2
1 BC(4,N)=1.0D0
1 BC(5,N)=1.0D0
1 BC(6,N)=1.0D0
RETURN
2 IF(SV-2.0D0) 3,3,4
2 BC(1,N)=1.0D0
2 BC(2,N)=1.0D0
2 BC(3,N)=1.0D0
RETURN
4 IF(SV-3.0D0) 5,5,6
4 BC(3,N)=1.0D0
4 BC(5,N)=1.0D0
4 BC(6,N)=1.0D0
RETURN
6 IF(SV-4.0D0) 7,7,8
6 BC(1,N)=1.0D0
6 BC(3,N)=1.0D0
6 BC(5,N)=1.0D0
RETURN
8 BC(2,N)=1.0D0
8 BC(3,N)=1.0D0
8 BC(6,N)=1.0D0
RETURN
END


```

SUBROUTINE STAVEO (SV,N,BC)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION BC(6,22)
DO 10 L=1,6
  BC(L,N)=0.0D0
10 IF(SV-1.0D0) 1,1,2
  1 BC(2,N)=1.0D0
  BC(5,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
2 IF(SV-2.0D0) 3,3,4
  3 BC(1,N)=1.0D0
  BC(3,N)=1.0D0
  BC(4,N)=1.0D0
  RETURN
4 IF(SV-3.0D0) 5,5,6
  5 BC(2,N)=1.0D0
  BC(4,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
6 IF(SV-4.0D0) 7,7,5
  7 BC(1,N)=1.0D0
  BC(4,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
END

```

010580
010590
010600
010610
010620
010630
010640
010650
010660
010670
010680
010690
010700
010710
010720
010730
010740
010750
010760
010770
010780
010790
010800
010810
010820
010830

```

SUBROUTINE INVERT (A,N,D,L,M)
IMPLICIT REAL*8(A-H,O-Z)
PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX
DIMENSION A(25,25),L(25),M(25)
SEARCH FOR LARGEST ELEMENT
D=1.0D0
DO 80 K=1,N
  L(K)=K
  M(K)=K
  BIGA=A(K,K)
DO 20 I=K,N
DO 20 J=K,N
  IF(DABS(BIGA)-DABS(A(I,J))) 10,20,20
10 BIGA=A(I,J)
  L(K)=I
  M(K)=J
20 CONTINUE
INTERCHANGE ROWS
J=L(K)
IF(L(K)-K) 35,35,25

```

010860
010870
010880
010890
010900
010910
010920
010930
010940
010950
010960
010970
010980
010990
011000
011010
011020
011030
011040
011050


```

25 DO 30 I=1,N
   HOLD=-A(K,I)
   A(K,I)=A(J,I)
30 A(J,I)=HOLD
   INTERCHANGE COLUMNS
35 I=M(K)
   IF(M(K)-K) 45,45,37
37 DO 40 J=1,N
   HOLD=-A(J,K)
   A(J,K)=A(J,I)
40 A(J,I)=HOLD
   DIVIDE COLUMN BY MINUS PIVOT
45 DO 55 I=1,N
   IF(I-K) 50,55,50
46 A(I,K)=A(I,K)/(-A(K,K))
50 CONTINUE
55 REDUCE MATRIX
   DO 65 I=1,N
   DO 65 J=1,N
56 IF(I-K) 57,65,57
57 IF(J-K) 60,65,60
60 A(I,J)=A(I,K)*A(K,J)+A(I,J)
65 CONTINUE
   DIVIDE ROW BY PIVOT
   DO 75 J=1,N
   IF(J-K) 70,75,70
70 A(K,J)=A(K,J)/A(K,K)
75 CONTINUE
   CONTINUED PRODUCT OF PIVOTS
   D=D*A(K,K)
   REPLACE PIVOT BY RECIPROCAL
   A(K,K)=1.000/A(K,K)
80 CONTINUE
   FINAL ROW AND COLUMN INTERCHANGE
   K=N
100 K=K-1
   IF(K) 150,150,103
103 I=L(K)
   IF(I-K) 120,120,105
105 DO 110 J=1,N
   HOLD=A(J,K)
   A(J,K)=-A(J,I)
   A(J,I)=HOLD
110 J=M(K)
120 IF(J-K) 100,100,125
125 DO 130 I=1,N
   HOLD=A(K,I)
   A(K,I)=-A(J,I)

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011060
011070
011080
011090
011100
011110
011120
011130
011140
011150
011160
011170
011180
011190
011200
011210
011220
011230
011240
011250
011260
011270
011280
011290
011300
011310
011320
011330
011340
011350
011360
011370
011380
011390
011400
011410
011420
011430
011440
011450
011460
011470
011480
011490
011500
011510
011520
011530

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```

130 A(J,I)=HOLD
140 GO TO 100
150 RETURN
END

```

```

011540
011550
011560
011570

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```

SUBROUTINE HANGER (CLX,CLY,CTZ,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
ZZ=1.0D0
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=ZZ
U(2,2)=ZZ
U(3,3)=ZZ
U(4,4)=ZZ
U(5,5)=ZZ
U(6,6)=ZZ
U(4,3)=CTZ
U(5,2)=-CLY
U(6,1)=CLX
RETURN
END

```

```

011600
011610
011620
011630
011640
011650
011660
011670
011680
011690
011700
011710
011720
011730
011740
011750
011760
011770

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```

SUBROUTINE HANGED (CTX,CLZ,CTY,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
ZZ=1.0D0
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=ZZ
U(2,1)=CTX
U(2,2)=ZZ
U(3,3)=ZZ
U(4,4)=ZZ
U(5,4)=CTY
U(5,5)=ZZ
U(6,3)=-CLZ
U(6,6)=ZZ
RETURN
END

```

```

011800
011810
011820
011830
011840
011850
011860
011870
011880
011890
011900
011910
011920
011930
011940
011950
011960
011970

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```

SUBROUTINE CFIELD(AJT,R,Z,G,SDRI,RHO,THETA,D,DI,FFAC,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
PHI=THETA/Z
RHO=RHO
IF(RHO-ZA) 11,11,12
11 V=-ZZ
GO TO 1
12 V=ZZ
1 CP=DCOS(PHI)
SP=DSIN(PHI)
W=ZZ/(G*AJT)
F1=(PHI*CP+SP)/2.0D0
F3=(SP-CP*PHI)/2.0D0
F5=(2.0D0-2.0D0*CP-PHI*SP)/2.0D0
F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)/2.0D0
RHO=DABS(RHO)
DO 2 J=1,6
DO 2 K=1,6
2 A(J,K)=ZA
13 IF(SDRI) 13,13,14
14 AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
SD=RHO*THETA/(G*Z*AREA)
GO TO 15
15 SD=ZA
A(1,1)=CP
A(1,2)=(W*RHO*F1)-(RHO*F3*R)
A(1,4)=V*SP
A(1,5)=V*(W+R)*(RHO*PHI*SP)/2.0D0
A(1,6)=V*(W+R)*(RHO**2)*F3
A(2,2)=CP
A(2,5)=V*SP
A(2,6)=V*RHO*(ZZ-CP)
A(3,1)=-A(2,6)
A(3,2)=-ZZ*A(1,6)
A(3,3)=ZZ
A(3,4)=RHO*SP
A(3,5)=(R*(RHO**2)*PHI*SP)/2.0D0-W*RHO**2*F5
A(3,6)=(R*(RHO**3*F3)-(W*F6*RHO**3)-SD
A(4,1)=-V*SP
A(4,2)=-V*(W+R)*RHO*PHI*SP/2.0D0
A(4,4)=CP
A(4,5)=(R*RHO*F1)-(W*RHO*F3)
A(4,6)=A(3,5)
A(5,2)=-V*SP
A(5,5)=CP

```

012000
012010
012020
012030
012040
012050
012060
012070
012080
012090
012100
012110
012120
012130
012140
012150
012160
012170
012180
012190
012200
012210
012220
012230
012240
012250
012260
012270
012280
012290
012300
012310
012320
012330
012340
012350
012360
012370
012380
012390
012400
012410
012420
012430
012440
012450
012460
012470

012480
012490
012500
012510
012520

A(5,6)=A(3,4)
A(6,6)=ZZ
RHO=RHOD
RETURN
END

012550
012560
012570
012580
012590
012600
012610
012620
012630
012640
012650
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012670
012680
012690
012700
012710
012720
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012740
012750
012760
012770

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SUBROUTINE POINO(AMU,AIX,AIY,OMEGA,TL,Z,SDRI,B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(6,6)
ZZ=1.0D0
ZA=0.0D0
AM=AMU*TL/(Z-ZZ)
DO 1 J=1,6
DO 1 K=1,6
1 B(J,K)=ZA
B(1,1)=ZZ
B(2,1)=-AM*(AIX*OMEGA)**2
B(2,2)=ZZ
B(3,3)=ZZ
B(4,4)=ZZ
IF(SDRI-ZA) 9,9,10
9 B(5,4)=-AM*(AIY*OMEGA)**2
GO TO 11
10 B(5,4)=ZA
11 B(5,5)=ZZ
B(6,3)=AM*OMEGA**2
B(6,6)=ZZ
RETURN
END

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012800
012810
012820
012830
012840
012850
012860
012870
012880
012890
012900
012910
012920
012930
012940
012950

```

SUBROUTINE CFIELD(R,Z,G,SD,RHO,THETA,D,DI,E,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
PHI=THETA/Z
RD=DABS(RHO)
IF(RHO)1,1,2
1 V=-ZZ
GO TO 3
2 V=ZZ
3 CP=DCOS(PHI)
SP=DSIN(PHI)
DO 4 J=1,6
DO 4 K=1,6
4 A(J,K)=ZA

```


012960
012970
012980
012990
013000
013010
013020
013030
013040
013050
013060
013070
013080
013090
013100
013110
013120
013130
013140
013150
013160
013170
013180
013190
013200
013210
013220
013230
013240
013250
013260

```
A(1,1)=CP
A(1,2)=-V*SP
A(1,3)=-V*RD*(ZZ-CP)
A(1,4)=-V*RD**2*R*(PHI-SP)
AR=3.141592654D0*(D**2-DI**2)/4.0D0
W=ZZ/(E*AR)
F3=(SP-PHI*CP)*0.50D0
F1=(PHI*CP+SP)*0.50D0
F5=(2.0D0-2.0D0*CP-PHI*SP)*0.50D0
F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)*0.50D0
A(1,5)=V*(RD*W*PHI*SP*0.5D0-R*F5*RD**3)
A(1,6)=RD*W*F1+R*F6*RD**3
A(2,1)=V*SP
A(2,2)=CP
A(2,3)=RD*SP
A(2,4)=R*RD**2*(ZZ-CP)
A(2,5)=RD*F3*(W+R*RD**2)
A(2,6)=A(1,5)
A(3,3)=ZZ
A(3,4)=RD*R*PHI
A(3,5)=A(2,4)
A(3,6)=A(1,4)
A(4,4)=ZZ
A(4,5)=A(2,3)
A(4,6)=A(1,3)
A(5,5)=CP
A(5,6)=A(1,2)
A(6,5)=A(2,1)
A(6,6)=CP
RETURN
END
```

SUBROUTINE POINT(AMU,AIY,OMEGA,TL,Z,RI,B)

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```
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(6,6)
ZZ=1.0D0
ZA=0.0D0
AM=AMU*TL/(Z-ZZ)
DO 1 J=1,6
DO 1 K=1,6
1 B(J,K)=ZA
B(1,1)=ZZ
B(2,2)=ZZ
B(3,3)=ZZ
IF(RI)2,2,3
2 B(4,3)=-AM*(AIY*OMEGA)**2
3 B(4,4)=ZZ
```


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B(5,2)=AM*OMEGA**2
B(5,5)=ZZ
B(6,1)=-B(5,2)
B(6,6)=ZZ
RETURN
END

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013870
013880

SUBROUTINE SFIELD(DI,TL,R,Z,G,SD,D,E,FFAC,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)
DL=TL/Z
DO 1 J=2,5
1 A(1,J)=ZA
A(1,1)=ZZ
A(1,6)=DL/(E*ARA)
A(2,1)=ZA
A(2,2)=ZZ
A(2,3)=DL**2*R/2.0D0
A(2,4)=DL**2*R/2.0D0
IF(SD) 2,2,3
2 A(2,5)=DL**3*R/6.0D0-DL/(G*AREA)
GO TO 4
3 A(2,5)=DL**3*R/6.0D0
4 A(2,6)=ZA
A(3,1)=ZA
A(3,2)=ZA
A(3,3)=ZZ
A(3,4)=DL*R
A(3,5)=A(2,4)
A(3,6)=ZA
DO 5 J=1,6
5 A(4,J)=ZA
A(5,J)=ZA
A(6,J)=ZA
A(4,4)=ZZ
A(4,5)=DL
A(5,5)=ZZ
A(6,6)=ZZ
RETURN
END


```

SUBROUTINE SFIELO(DI,TL,AJT,R,Z,G,SDRI,D,FFAC,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
DL=TL/Z
DO 2 J=1,6
DO 2 K=1,6
2 A(J,K)=ZA
A(1,1)=ZZ
A(2,2)=ZZ
A(3,3)=ZZ
A(4,4)=ZZ
A(5,5)=ZZ
A(6,6)=ZZ
IF(SDRI-ZA) 7,7,8
7 A(3,6)=DL**3*R/6.0D0-DL/(G*AREA)
GO TO 1
8 A(3,6)=DL**3*R/6.0D0
1 A(1,2)=DL/(G*AJT)
A(3,4)=DL
A(3,5)=DL**2*R/2.0D0
A(4,5)=DL*R
A(4,6)=A(3,5)
A(5,6)=DL
RETURN
END

```

```

SUBROUTINE MATMUL(A,B,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6),B(6,6),C(6,6),U(6,6)
DO 1 J=1,6
DO 1 K=1,6
C(J,K)=0.0D0
DO 1 L=1,6
1 C(J,K)=C(J,K)+B(J,L)*A(L,K)
DO 2 J=1,6
DO 2 K=1,6
2 U(J,K)=C(J,K)
RETURN
END

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013910
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014000
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014180

014210
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014240
014250
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014270
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014290
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```

SUBROUTINE FINBRA(U,VV,V,MAT,A,Z)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6),A(6,6),VV(6,3),V(6,3)
IF(MAT) 65,64,65
64 DO 67 J=1,6
   DO 67 K=1,3
     V(J,K)=0.0D0
67 DO 67 L=1,6
   V(J,K)=V(J,K)+U(J,L)*VV(L,K)
65 RETURN
N=Z-1.0D0
DO 5 M=1,N
DO 4 J=1,6
DO 4 K=1,3
  V(J,K)=0.0D0
4 DO 4 L=1,6
  V(J,K)=V(J,K)+U(J,L)*VV(L,K)
DO 5 J=1,6
DO 5 K=1,3
  VV(J,K)=V(J,K)
5 DO 7 J=1,6
  DO 7 K=1,3
    V(J,K)=0.0D0
DO 7 L=1,6
  V(J,K)=V(J,K)+A(J,L)*VV(L,K)
7 RETURN
END

```

014360
014370
014380
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014600
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```

SUBROUTINE FINMAT(U,UU,V,MAT,A,Z)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6),A(6,6),V(6,3),UU(6,3)
ZA=0.0D0
IF(MAT) 65,64,65
64 DO 67 J=1,6
   DO 67 K=1,3
     V(J,K)=ZA
67 DO 67 L=1,6
   V(J,K)=V(J,K)+U(J,L)*UU(L,K)
65 RETURN
N=Z-1.0D0
DO 5 M=1,N
DO 4 J=1,6
DO 4 K=1,3
  V(J,K)=ZA
4 DO 4 L=1,6
  V(J,K)=V(J,K)+U(J,L)*UU(L,K)
DO 5 J=1,6

```

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```

5 DO 5 K=1,3
  UU(J,K)=V(J,K)
  DO 7 J=1,6
    DO 7 K=1,3
      V(J,K)=ZA
    DO 7 L=1,6
      V(J,K)=V(J,K)+A(J,L)*UU(L,K)
  7 RETURN
  END

```

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```

SUBROUTINE BRANCH(R3,VV,PHI,U)
  IMPLICIT REAL*8(A-H,O-Z)
  DIMENSION R3(3,3),VV(6,3),U(6,6),R1(25,25),LL(25),MM(25),R2(3,3),
  1 R(3,3),G1(3,3),G2(3,3)
  ZZ=1.0D0
  ZA=0.0D0
  DO 4 L=1,3
    R1(1,L)=VV(1,L)
    R1(2,L)=VV(2,L)
    R1(3,L)=VV(3,L)
    R2(1,L)=VV(4,L)
    R2(2,L)=VV(5,L)
    R2(3,L)=VV(6,L)
  4 SP=DSIN(PHI)
  CP=DCOS(PHI)
  G1(1,1)=CP
  G1(1,2)=-SP
  G1(1,3)=ZA
  G1(2,1)=SP
  G1(2,2)=CP
  G1(2,3)=ZA
  G1(3,1)=ZA
  G1(3,2)=ZA
  G1(3,3)=ZZ
  G2(1,1)=ZA
  G2(1,2)=ZA
  G2(1,3)=ZA
  G2(2,1)=CP
  G2(2,2)=-SP
  G2(2,3)=ZA
  G2(3,1)=SP
  G2(3,2)=CP
  G2(3,3)=ZA
  CALL INVERT (R1,3,D,LL,MM)
  DO 5 J=1,3
    DO 5 K=1,3
      R3(J,K)=ZA

```



```

5 DO 5 L=1,3
  R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
DO 6 J=1,3
  DO 6 K=1,3
    R1(J,K)=ZA
DO 6 L=1,3
  R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
DO 7 J=1,3
  DO 7 K=1,3
    R(J,K)=ZA
DO 7 L=1,3
  R(J,K)=R(J,K)+R1(J,L)*R3(L,K)
DO 8 J=1,6
  DO 8 K=1,6
    U(J,K)=ZA
DO 8 L=1,6
  DO 8 K=1,6
    U(1,L)=ZZ
    U(2,L)=ZZ
    U(3,L)=ZZ
    U(4,L)=ZZ
    U(5,L)=ZZ
    U(6,L)=ZZ
DO 9 L=1,3
  U(4,L)=R(1,L)
  U(5,L)=R(2,L)
  U(6,L)=R(3,L)
9 RETURN
END

```

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```

SUBROUTINE BRANCO(R3,VV,PHI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R3(3,3),U(6,6),VV(6,3),G1(3,3),G2(3,3)
1, S(3,3),LL(25),MM(25)
ZZ=1.ODO
ZA=0.ODO
DO 4 K=1,3
  R1(1,K)=VV(1,K)
  R1(2,K)=VV(3,K)
  R1(3,K)=VV(4,K)
  R2(1,K)=VV(2,K)
  R2(2,K)=VV(5,K)
  R2(3,K)=VV(6,K)
4 SP=DSIN(PHI)
CP=DCOS(PHI)
G1(1,1)=CP
G1(1,2)=ZA
G1(1,3)=-SP
G1(2,1)=ZA

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016270

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G1(2,2)=ZZ
G1(2,3)=ZA
G1(3,1)=SP
G1(3,2)=ZA
G1(3,3)=CP
G2(1,1)=CP
G2(1,2)=SP
G2(1,3)=ZAp
G2(2,1)=CP
G2(2,2)=ZA
G2(2,3)=ZA
G2(3,1)=ZA
G2(3,2)=ZZ
G2(3,3)=ZZ
CALL INVERT (R1,3,D,LL,MM)
DO 5 J=1,3
DO 5 K=1,3
R3(J,K)=ZA
DO 5 L=1,3
R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
5 DO 6 J=1,3
DO 6 K=1,3
R1(J,K)=ZA
DO 6 L=1,3
R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
6 DO 7 J=1,3
DO 7 K=1,3
S(J,K)=ZA
DO 7 L=1,3
S(J,K)=S(J,K)+R1(J,L)*R3(L,K)
7 DO 8 J=1,6
DO 8 K=1,6
U(J,K)=ZA
U(1,1)=ZZ
U(2,1)=S(1,1)
U(2,2)=ZZ
U(2,3)=S(1,2)
U(2,4)=S(1,3)
U(3,3)=ZZ
U(4,4)=ZZ
U(5,1)=S(2,1)
U(5,3)=S(2,2)
U(5,4)=S(2,3)
U(5,5)=ZZ
U(6,1)=S(3,1)
U(6,3)=S(3,2)
U(6,4)=S(3,3)
U(6,6)=ZZ

```


RETURN
END

```

SUBROUTINE STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION SVI(6,1),UU(6,3)
ZZ=1.0D0
ZA=0.0D0
M=0
DO 1 J=1,6
DO 1 K=1,3
1 UU(J,K)=ZA
40 IF(PM) 40,40,41
40 DO 42 J=1,6
43 IF(SVI(J,1)) 42,42,43
43 M=M+1
UU(J,M)=ZZ
IF(M-1) 42,44,42
44 UU(J,2)=-D1
42 CONTINUE
41 P1=P1+D1
P2=P2+D2
IF(SS)14,15,15
15 IF(SS-ZZ) 12,11,10
14 IF(PP-ZZ) 12,11,10
10 DO 4 J=1,6
4 IF(SVI(J,1)) 4,4,2
2 M=M+1
UU(J,M)=ZZ
IF(M-2) 4,3,5
3 UU(J,1)=P1
GO TO 4
5 UU(J,1)=P2
4 CONTINUE
RETURN
11 DO 21 J=1,6
22 IF(SVI(J,1))21,21,22
22 M=M+1
UU(J,M)=ZZ
IF(M-1) 25,26,25
26 UU(J,3)=P1
25 IF(M-2) 21,23,21
23 UU(J,3)=P2
21 CONTINUE
RETURN

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016280
016290

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016690
016700
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016840

```
12 DO 31 J=1,6
   IF(SVI(J,1)) 31,31,32
32 M=M+1
   UU(J,M)=ZZ
   IF(M-1) 35,36,35
36 UU(J,2)=P2
35 IF(M-2) 31,31,33
33 UU(J,2)=P1
31 CONTINUE
   RETURN
   END
```

016870
016880
016890
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017000
017005
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017200

```
SUBROUTINE STATEB(SVB,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION VV(6,3),YY(3,3,25),V(3,3),SVB(6,22)
ZZ=1.0D0
ZA=0.0D0
B1=ZA
B2=ZA
B3=ZA
D1=ZA
D2=ZA
IF(BRC) 33,40,22
IF(L-1) 23,20,21
IF(L-1) 23,20,23
DO 10 J=1,3
DO 10 K=1,3
V(J,K)=YY(J,K,N)
IF(SS) 11,12,12
IF(PP-ZZ) 33,31,30
IF(SS-ZZ) 33,31,30
B1=V(1,1)+V(1,2)*D1+V(1,3)*D2+B1
B2=V(2,1)+V(2,2)*D1+V(2,3)*D2+B2
B3=V(3,1)+V(3,2)*D1+V(3,3)*D2+B3
GO TO 33
B1=B1+V(1,1)*D2+V(1,3)*D1+V(1,2)
B2=B2+V(2,1)*D2+V(2,3)*D1+V(2,2)
B3=B3+V(3,1)*D2+V(3,3)*D1+V(3,2)
GO TO 33
B1=B1+V(1,1)*D1+V(1,2)*D2+V(1,3)
B2=B2+V(2,1)*D1+V(2,2)*D2+V(2,3)
B3=B3+V(3,1)*D1+V(3,2)*D2+V(3,3)
Q1=Q1*B1
Q2=Q1*B2
Q3=Q1*B3
M=1
DO 1 J=1,6
```



```

1  DO 1 K=1,3
   VV(J,K)=ZA
2  DO 4 J=1,6
   IF(SVB(J,N)) 4,4,2
3  IF(M-1) 7,3,7
35 IF(PM) 34,35,35
   VV(J,1)=Q1
   GO TO 36
34 VV(J,1)=ZZ
36 M=M+1
   GO TO 4
7  IF(M-2) 4,8,9
8  VV(J,1)=Q2
   VV(J,M)=ZZ
   M=M+1
   GO TO 4
9  VV(J,1)=Q3
   VV(J,M)=ZZ
4  CONTINUE
   RETURN
40 IF(FINK) 54,51,54
54 IF(BRC) 51,51,52
52 DO 53 J=1,3
   DO 53 K=1,3
53  V(J,K)=YY(J,K,N)
   D1=(V(1,2)/V(1,1)+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.0D0
   D2=(V(1,3)/V(1,1)+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.0D0
51  M=0
   DO 41 J=1,6
   DO 41 K=1,3
   VV(J,K)=ZA
41  DO 42 J=1,6
   IF(SVB(J,N)) 42,42,43
43  M=M+1
   VV(J,M)=ZZ
   IF(M-1) 42,44,42
44  VV(J,2)=-D1
   VV(J,3)=-D2
42  CONTINUE
   RETURN
   END

```

```

SUBROUTINE BRCOR(R3,UU,XX,JK,VV,PM)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R3(3,3),UU(6,3),XX(3,3),UUU(3,3),VV(6,3)
ZZ=1.0D0
ZA=0.0D0

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017600
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017640
017650
017660
017670

```



```

20 IF(PM) 20,21,20
10 IF(JK) 10,10,11
00 12 J=1,3
00 12 K=1,3
12 UUU(J,K)=UU(J,K)
11 GO TO 3
00 13 K=1,3
UUU(1,K)=UU(1,K)
UUU(2,K)=UU(3,K)
13 UUU(3,K)=UU(4,K)
00 1 J=1,3
00 1 K=1,3
XX(J,K)=ZA
00 1 L=1,3
1 XX(J,K)=XX(J,K)+1
RETURN
21 IF(JK) 25,25,22
25 23 J=1,3
00 23 K=1,3
23 XX(J,K)=VV(J,K)
RETURN
22 24 K=1,3
XX(1,K)=VV(1,K)
XX(2,K)=VV(3,K)
24 XX(3,K)=VV(4,K)
RETURN
END

```

```

SUBROUTINE DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,
1PM,D1,D2)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION UU(6,3),V(6,3),SVE(6,1),D(3,3)
ZZ=1.0D0
ZA=0.0D0
L=0
X1=ZA
X2=ZA
Y2=ZA
Y1=ZA
D1=ZA
D2=ZA
DO 111 J=1,3
DO 111 K=1,3
D(J,K)=ZA
111
DO 91 N=1,6
IF(SVE(N,1)) 91,2,91
2 L=L+1

```



```

DO 92 K=1,3
V(L,K)=UU(N,K)
92 CONTINUE
91 IF(PM) 66,64,61
64 IF(FINK) 63,62,63
62 D1=(V(1,2)/V(1,1))+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.000
D2=(V(1,3)/V(1,1))+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.000
FINK=ZZ
RETURN
63 FINK=ZA
RETURN
66 RETURN
61 IF(SS) 311,312,312,400,401,402
311 IF(PP-ZZ) 313,314,315
312 IF(SS-ZZ) 300,403,300
402 IF(RR-ZZ) 340,341,340
315 IF(RR-ZZ) 340,341,340
300 IF(V(3,3)) 93,94,93
340 IF(V(3,2)) 93,94,93
93 DO 95 J=2,3
DO 95 K=1,3
D(J,K)=V(J,K)
95 DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
IF(DV) 98,94,98
94 RR=ZZ
HH=ZZ
RETURN
403 IF(V(2,3)) 96,200,96
341 IF(V(2,2)) 96,200,96
96 DO 97 K=1,3
D(2,K)=V(3,K)
97 D(3,K)=V(2,K)
DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
IF(DV) 98,200,98
98 X1=X1-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV
X2=X2-(V(1,2)*D(2,1)-V(1,1)*D(2,2))/DV
IF(X1) 100,102,100
IF(X2) 100,101,100
102 FF=ZZ
101 RETURN
100 IF(SS) 354,355,355
354 Y2=Y2-D(3,1)/D(3,3)-X1*D(3,2)/D(3,3)
RETURN
355 Y1=Y1-D(3,1)/D(3,2)-X2*D(3,3)/D(3,2)
RETURN
99 IF(SS) 20,21,21
20 PP=ZZ
GO TO 23

```


21	SS=ZZ				018640
23	RR=ZA				018650
	HH=ZZ				018660
	RETURN				018670
314	IF(RR-ZZ)	330,331,330			018680
401	IF(RR-ZZ)	301,404,301			018690
330	IF(V(3,1))	81,82,81			018700
301	IF(V(3,2))	81,82,81			018710
81	DO 83	J=2,3			018720
	DO 83	K=1,3			018730
83	DO(J,K)	=V(J,K)			018740
	DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)				018750
	IF(DV)	86,82,86			018760
82	RR=ZZ				018770
	HH=ZZ				018780
	RETURN				018790
331	IF(V(2,1))	84,70,84			018800
404	IF(V(2,2))	84,70,84			018810
84	DO 85	K=1,3			018820
	DO(2,K)	=V(3,K)			018830
85	DO(3,K)	=V(2,K)			018840
	DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)				018850
	IF(DV)	86,70,86			018860
86	X1=X1-(V(1,3)*D(2,2)-V(1,2)*D(2,3))/DV				018870
	X2=X2-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV				018880
	IF(X1)	203,201,203			018890
201	IF(X2)	203,202,203			018900
202	FF=ZZ				018910
	RETURN				018920
203	IF(SS)	352,353,353			018930
353	Y1=Y1-D(3,3)/D(3,1)-X2*D(3,2)/D(3,1)				018940
	RETURN				018950
352	Y2=Y2-D(3,3)/D(3,2)-X1*D(3,1)/D(3,2)				018960
	RETURN				018970
70	IF(SS)	25,24,24			018980
24	SS=2.0D0				018990
	GO TO 26				019000
25	PP=2.0D0				019010
26	RR=ZA				019020
	HH=ZZ				019030
	RETURN				019040
313	IF(RR-ZZ)	320,321,320			019050
400	IF(RR-ZZ)	302,405,302			019060
320	IF(V(3,3))	71,72,71			019070
302	IF(V(3,1))	71,72,71			019080
71	DO 73	J=2,3			019090
	DO 73	K=1,3			019100
73	D(J,K)	=V(J,K)			019110


```

DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
IF(DV) 76,72,76
72 RR=ZZ
   HH=ZZ
   RETURN
405 IF(V(2,1)) 74,99,74
321 IF(V(2,3)) 74,99,74
200 KKK=1
   RR=ZA
   RETURN
74 DO 75 K=1,3
   D(2,K)=V(3,K)
75 D(3,K)=V(2,K)
   DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
   IF(DV) 76,99,76
76 X2=X2-(V(1,2)*D(2,3)-V(1,3)*D(2,2))/DV
   X1=X1-(V(1,1)*D(2,2)-D(2,1)*V(1,2))/DV
211 IF(X1) 213,211,213
212 IF(X2) 213,212,213
   FF=ZZ
   RETURN
213 IF(SS) 350,351,351
351 Y1=Y1-(D(3,2)+X2*D(3,1))/D(3,3)
   RETURN
350 Y2=Y2-(D(3,2)+X1*D(3,3))/D(3,1)
   RETURN
END

```

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019120
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13. ABSTRACT

An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotatory inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.

Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.

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